HYP2026

20th International Conference on

Hyperbolic Problems: Theory, Numerics and Applications (HYP2026)

Stuttgart
May 25-29, 2026

General Information

Book of Abstracts

University of Stuttgart
Germany

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

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WELCOME ADDRESS FROM THE CONFERENCE CHAIRS

We are delighted to welcome you to HYP2026 in Stuttgart!

The HYP conference series on hyperbolic problems was first held 1986 in St. Etienne (France), and has been held every two years since then: 1988 Aachen (Germany), 1990 Uppsala (Sweden), 1992 Taormina (Italy), 1994 Stony Brook (USA), 1996 Hong Kong (China), 1998 Zürich (Switzerland), 2000 Magdeburg (Germany), 2002 Pasadena (USA), 2004 Osaka (Japan), 2006 Lyon (France), 2008 College Park, Maryland (USA), 2010 Beijing (China), 2012 Padova (Italy), 2014 Rio de Janeiro (Brazil), 2016 Aachen (Germany), 2018 University Park, Pennsylvania (USA), 2022 Málaga (Spain), and 2024 Shanghai (China). This year, the twentieth edition of the HYP2026 conference will be held on the city center campus of the University of Stuttgart.

In keeping with the spirit of the series, the main objective of HYP2026 is to bring together researchers and students with an interest in the theoretical, numerical and application aspects of hyperbolic PDEs and related models. The HYP2026 conference will provide an inspiring and productive forum for getting acquainted with the most recent developments in hyperbolic problems, stimulating and exchanging new ideas from different disciplines, and formulating new, challenging problems that will impact diverse fields of application. The conference aims to maintain the traditional balance of the HYP series by blending theory, numerics and applicationsIt comprises a wide range of topics including

- Theory and applications of hyperbolic balance laws
- Euler and Navier-Stokes equations, other PDEs in fluid dynamics
- Nonlinear wave equations, relativistic motion, and general relativity
- Kinetic equations and scaling limits of interacting particle systems
- Multifield problems, mixed-type PDEs, and PDEs of mathematical physics
- Stochastic evolution equations of hyperbolic type
- Multiscale problems and asymptotic modeling
- Numerical methods and scientific computing
- Optimization, control problems and inverse modeling
- Machine learning and data science aspects of hyperbolic conservation laws

The scientific program consists of five 50-minute plenary lectures and nine 45-minute invited lectures held in parallel. Moreover, there are seven parallel sessions with contributed lectures of 25 minutes. Particular highlights are the HYP2026 prize lectures. The Scientific Committee has decided to award the James Glimm Lecture to Eduard Feireisl in recognition of his fundamental and original contributions to mathematical fluid mechanics. The Peter Lax Award will be presented to Ruiwen Shu in honour of his early career achievements in the field. It is awarded to outstanding young researchers (10 years within the PhD). The award ceremonies and accompanying lectures will take place at Weißer Saal in the Neues Schloss, the residence of the former dukes and kings of Württemberg.

We thank all members of the University of Stuttgart and Johannes Gutenberg University Mainz that helped in the organization of the event, foremost the organizing committee with Andrea Beck, Aaron Brunk, Katja Stefanie Engstler, Maximilian Hörl, Jens Keim, Sabine Sämisch, Florian Wendt and above all Sylvia Zur.

Very special thanks go to Alina Chertock (North Carolina State University) who generously provides support for participants from the United States of America.

Your HYP2026 conference chairs,

Mária Lukácová-Medvidová

Johannes Gutenberg
University Mainz

Christian Rohde

University of Stuttgart

We gratefully acknowledge the support of the following institutions enabling us keep the conference fee for all participants to a minimum. Their contributions are greatly appreciated.

World Scientific
Connecting Great Minds

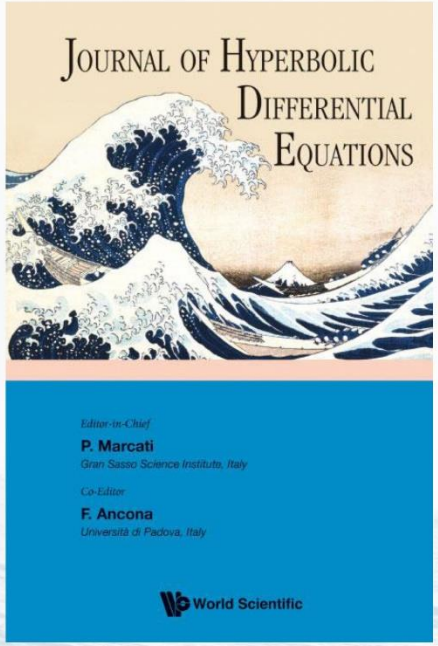
World Scientific Connect

Journal of Hyperbolic Differential Equations (JHDE)

- **Scope & Focus**
  - Publishes original research on nonlinear hyperbolic problems and related areas in mathematics and physics, including hyperbolic conservation laws, partial differential equations, mathematical physics, Lorentzian geometry, fluid dynamics, and numerical methods for hyperbolic equations

- **Impact & Reach**
  - 2024 Impact Factor: **1.1**
  - 2024 CiteScore: **1.3**
  - **Ranked Q2 in Mathematics, Applied (Web of Science)**
  - Readership (2023–2025): **45,000**

- **Featured Special Issues**
  - **Special Issue on Control Methods in Hyperbolic PDEs (2026)**
    - Guest Editors: Fabio Ancona, Olivier Glass and Michael Herty
  - **Special Issue: Recent Advances in Nonlinear Hyperbolic PDEs and Fluid Dynamics (2024)**
    - Special Issue Editors: Fabio Ancona, Stefano Bianchini



SCIENTIFIC COMMITTEE OF HYP2026

Remi Abgrall, University of Zurich (Switzerland)

Sylvie Benzoni, Université Claude Bernard, Lyon 1(France)

Alberto Bressan, Penn State, Eberly College of Science (USA)

Raimund Bürger, Universidad de Concepción (Chile)

Gui-Qiang G. Chen, University of Oxford (United Kingdom)

Michael Dumbser, University of Trento (Italy)

Eduard Feireisl, Czech Academy of Sciences (Czech Republic)

Helge Holden, Norwegian University of Science and Technology NTNU (Norway)

Song Jiang, Institute of Applied Physics and Computational Mathematics (PR China)

Shi Jin, Shanghai Jiao Tong University (PR China)

Christian Klingenberg, University of Würzburg (Germany)

Mária Lukácová-Medvidová, Johannes Gutenberg University Mainz (Germany)

Carlos Parés Madroñal, University of Málaga (Spain)

Gabriella Puppo, Sapienza Università di Roma (Italy)

Chi-Wang Shu, Brown University, Providence (USA)

Eitan Tadmor, University of Maryland, College Park (USA)

PRACTICAL INFORMATION

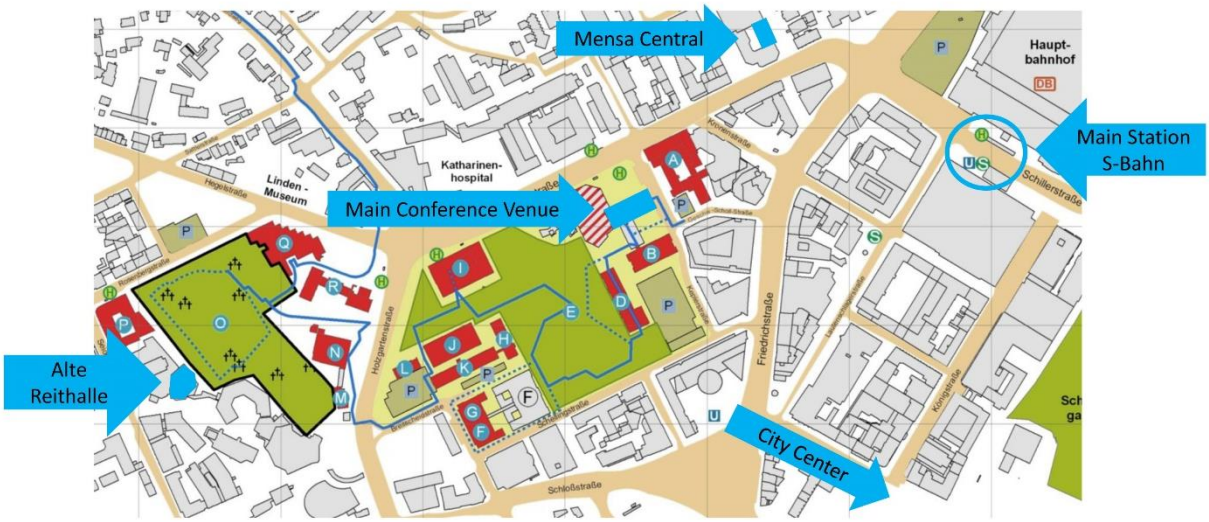
Locations and Directions

The Main Conference Venue is the **Kollegiengebäude II (KII).** All lectures from Monday, May 25 – Thursday, May 28 take place in KII.

Address: Keplerstraße 17, 70174 Stuttgart

From Stuttgart Main Station check this link.
The figure displays the view from Stadtgarten park. KII is the building on the left.

The Info Point with the registration is located in the foyer of KII.



Campus plan and surroundings.

The scientific program on Friday, May 29, takes place at **Neues Schloss Stuttgart**. It includes the award lectures and the final plenary lecture.

Address:
Schloßplatz 4, 70173 Stuttgart

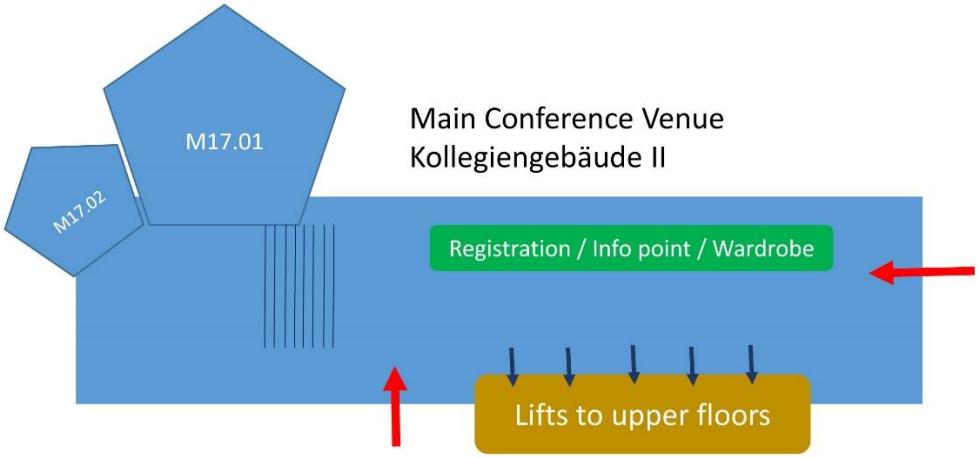
The Neues Schloss is the former residence of the dukes and kings of Württemberg. The three-winged complex was built in the mid of the eighteenth century as one of the last baroque-style residences in Europe. It forms the final section of the buildings surrounding the central castle square.

Photo: Katja Stefanie Engstler

Registration & Info Point

When first arriving at the conference, please check in at the **Info Point** in the foyer of KII, to pick up your name badge and conference bag. Registration will be possible from 8:30 to 18:30 from Monday, May 25 – Thursday, May 28.

If you have any questions, the conference staff (with orange-coloured T-shirts) at the Info Point in the foyer and in the lecture halls will help you. Feel free to contact them.



Map of the foyer: main lecture halls and access to rooms for contributed sessions.

You may leave your jackets and luggage at a wardrobe in the foyer. It is open during the opening times of the Info Point. There will be a wardrobe in Neues Schloss on Friday, May 29.

We do not provide a printed certificate of attendance. After the end of the conference you can download it from your conftool account.

WLAN

There is WiFi connectivity via eduroam for registered eduroam users on the campus. Alternatively, you can connect via the free access WLAN-ESSID uni-stuttgart-open. Please scan the QR codes:

By using uni-stuttgart-open, you agree with the User Regulations for Digital Information Processing and Communication Equipment (IdC) at the University of Stuttgart.

Lunch Breaks

Registered members can use their name badge to get free lunches at the **Mensa Central** (see Campus Plan) of the University of Stuttgart. You can choose your menu (1 plate of food, side dishes and 1 soft drink) and then simply show your badge at the cashiers when checking out.

Monday, May 25, is a public holiday in Germany. The Mensa Central is closed and we serve a **light meal** in the foyer of KII.

There are multiple alternatives of all kinds around the Main Conference Venue including a small cafeteria in KII (closed on Monday, May 25) and the Mezzogiorno Restaurant right next to KII in the Stadtgarten.

The following Swabian brewery-style restaurants are quite large, serve lunch, and are within a 5-10 minute walk in direction of the city center:

- Wirtshaus zum Kesselwirt (Lautenschlagerstr. 24)
- Brauhaus Schönbuch (Bolzstr. 10)
- Carls Brauhaus (Schlossplatz)
- Alte Kanzlei Stuttgart (Schlossplatz)

Social Activities

Welcome Reception:

On Monday, May 25, we invite you for a welcome reception in the foyer of KII, starting from 18:15 on.

Photos and Conference Photo:

Please note, that there is a photographer during the conference. The photos will be published according to our terms and conditions.

The conference photo is planned for Tuesday, May 26, after the first morning session (approximately at 10:50).

Excursions:

For Wednesday afternoon we offer some guided tours in Stuttgart including a coach city tour and some more or less heavy walking tours. You can make your choice on conftool according to first-come-first-serve.

Conference Dinner:

The conference dinner on Thursday, May 28, is to be held at **Alte Reithalle.** If not yet done via conftool, you can register for the conference dinner at the Info Point.

Address:
Forststraße 2a, 70174 Stuttgart

The Alte Reithalle is close to the Maritim Hotel and within walking distance from the Main Conference Venue KII.

The iron-and-glass structure, built in the neo-renaissance style, was constructed by an architect of the University of Stuttgart at the end of the nineteenth century. As one of the first multipurpose halls it was intended to be used for equestrian events, sport shows, and circus performances.

Photo: Wikipedia

Photo: Maritim Hotel Stuttgart

The dinner speech will be held by Christian Klingenberg (University of Würzburg) on

Hyperbolic equations since Glimm — the perspective of a contemporary witness

Abstract: I will outline the development of the subject of hyperbolic equations as I observed it in my academic life. The series of HYP conferences since the first one in 1986 have mirrored this evolution. At the national level, this development has taken different forms. This presentation is dedicated to the memory of two researchers, who passed away since HYP2024: Peter D. Lax and Philip L. Roe.

Conference Proceedings

The conference proceedings of HYP2026 will be published by Springer Proceedings in Mathematics & Statistics.

https://link.springer.com/series/10533

The submission of papers will be opened in due time after the conference has been held. Submissions can be done upon invitation by the editors. All papers undergo a peer-review process.

THE SCIENTIFIC PROGRAM

Access to Scientific Program

1. On the **HYP2006 website** you find the program details and the Book of Abstracts. The website accounts for the latest changes.

2. The **Conference4me** smartphone app provides participants with a convenient tool for planning their participation at HYP2026. The app is available for Android, iOS, and Windows Phone.

   To download the app, please visit the app store on your mobile device or scan the QR code provided below. After installation, search for “HYP2026” to access the conference. You will then be able to browse the complete program directly from your phone or tablet and create your own personalized agenda.

   Please make sure to regularly update the conference agenda to stay informed about any changes
   (Menu → Program → three dots in the top-right corner → Update Agenda).

3. We will display a printed program on a board in the Foyer of KII.

Program at a glance

		Monday, May 25		Tuesday, May 26		
		8:30 – 9:45	Registration	8:30 – 9:00	Registration	
Morning Sessions (KII)		9:45 – 10:00	Opening	9:00 – 9:50	Sigal Gottlieb	
		10:00 – 10:50	Vlad Vicol	9:55 – 10:35	Bangwei She (M17-01)	Li Wang (M17-02)
		Coffee Break in KII Foyer		Coffee Break in KII Foyer with Conference Photo		
		11:20 – 12:00	Lorenzo Pareschi	11:20 – 12:00	Kailiang Wu (M17.01)	Elia Brue (M17.02)
12:00-14:00		Lunch Break				
Afternoon Sessions (KII)	14:00 – 14:25	Contributed Lectures		Contributed Lectures		
	14:25 – 14:50					
	14:50 – 15:15					
	15:15 – 15:40					
		Coffee Break in KII Foyer				
	16:00 – 16:25	Contributed Lectures		Contributed Lectures		
	16:25 – 16:50					
	16:50 – 17:15					
	17:15 – 17:40					
	17:40 – 18:05					
		Welcome Reception in KII Foyer, starts at 18:10				

		Wednesday, May 27			Thursday, May 28		
		8:30 – 9:00	Registration		8:30 – 9:00	Registration	
Morning Sessions (KII)		9:00 – 9:50	Manuel Castro		9:00 – 9:50	Francis Filbet	
		9:55 – 10:35	Michael Herty (M17.01)	Gianluca Crippa (M17.02)	9:55 – 10:35	Gregor Gassner (M17.01)	Katrin Grunert (M17.02)
		Coffee Break in KII Foyer					
		11:10 – 12:00	Gabriella Puppo		11:10 – 11:35	Contributed Lectures	
					11:35 – 12:00		
12:00-14:00		Lunch Break					
Afternoon Sessions (KII)	14:00 – 14:25	Free Discussions and Excursions			Contributed Lectures		
	14:25 – 14:50						
	14:50 – 15:15						
	15:15 – 15:40						
					Coffee Break in KII Foyer		
	16:00 – 16:25				Contributed Lectures		
	16:25 – 16:50						
	16:50 – 17:15						
	17:15 – 17:40						
	17:40 – 18:05						
					Conference Dinner in "Alte Reithalle", starts at 19:30		

	Friday, May 29	
Neues Schloss Stuttgart "Weißer Saal"	9:00 – 9:50	James Glimm Lecture Eduard Feireisl
	9:50 – 10:40	Peter Lax Award Lecture Ruiwen Shu
	Coffee Break	
	11:10 – 12:00	Gui-Qiang Chen
	12:00 – 12:15	Closing

Conference Halls

Monday, May 25 – Thursday May 28: All plenary/invited lectures and some of the contributed lectures take place in the lecture halls M17.01 and M17.02 in the ground floor of KII, see the map of the foyer. Most of the contributed lectures are held in rooms in the upper floors of KII. In the detailed program these rooms are identified by some ID M17.XX and a room number:

The first entry of the room number indicates the floor:

2.009
second floor – room 9

On each floor will be signs that lead to the specific rooms.

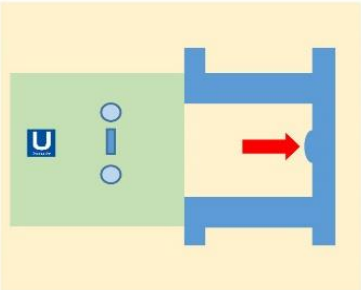
These rooms can be reached via the staircase or the lifts in the foyer of KII. Please note the usage of the lifts:

First enter your destination floor on the terminals in front of the lift.

The terminal display now will show you the number of the lift that brings you to the desired floor. It is not possible to change the floor from inside the lift.

Friday, May 29: All lectures take place in the hall **Weißer Saal** at **Neues Schloss.**

For **Weißer Saal** you enter the castle through the main entrance from the Schlossplatz right in the middle of the building opposite to the iconic Jubiläumssäule.



Instructions for speakers

Please check the time and lecture room of your presentation in the daily program and on the info boards as there might have been changes.

A session helper is assigned to each lecture room for help with technical equipment. Each lecture room is equipped with a computer (Microsoft Windows, Microsoft Office, Acrobat Reader) and a beamer. Your slides shall be prepared in the format of 16:9, whereas 4:3 is also possible.

You are asked to upload your presentation at the very latest in the break before the session to the session computer via an USB stick (available from the session helpers). Please be present at least 15 minutes prior to the start of your session and let the chairperson know you are there.

The time scheduled for lectures is

- 25 min. (incl. discussion) for contributed lectures,
- 40 min. (incl. discussion) for invited lectures,
- 50 min. (incl. discussion) for plenary lectures.

The chairpersons are requested to stop presentations after the scheduled time has passed.

Prize Lectures, Plenary and Invited speakers

James Glimm Lecture

Eduard Feireisl (Czech Academy of Sciences, Czech Republic)
From real to ideal compressible fluids
Friday, May 29: 9:00 - 9:50, Location: Weißer Saal (Neues Schloss)

Peter Lax Award Lecture

Ruiwen Shu (University of Georgia, USA)
Recent progress on interaction energy minimizers
Friday, May 29: 9:50 - 10:40, Location: Weißer Saal (Neues Schloss)

Elia Bruè (Bocconi University, Italy)
Two-dimensional Euler flows beyond Yudovich
Tuesday, May 26: 11:20 - 12:00, Location: M17.02 (KII)

Gui-Qiang Chen (University of Oxford, United Kingdom)
Multidimensional Riemann Problems and Hyperbolic Conservation Laws
Friday, May 29: 11:10 - 12:00, Location: Weißer Saal (Neues Schloss)

Gianluca Crippa (University of Basel, Switzerland)
Singular limits in some nonlinear continuity equations
Wednesday, May 27: 9:55 - 10:35, Location: M17.02 (KII)

Manuel J. Castro Díaz (University of Málaga, Spain)
High-Order Well-Balanced Schemes for Balance Laws: Recent Advances, Challenges, and Numerical Results
Wednesday, May 27: 9:00 - 9:50, Location: M17.01 (KII)

Francis Filbet (Université de Toulouse III, France)
Structure and asymptotic preserving schemes for the Vlasov-Poisson system and related models
Thursday, May 28: 9:00 - 9:50, Location: M17.01 (KII)

Gregor Gassner (University of Cologne, Germany)
Recent Advances in Subcell Based Limiting for Discontinuous Galerkin Methods with Guarantees
Thursday, May 28: 9:55 - 10:35, Location: M17.01 (KII)

Sigal Gottlieb (University of Massachusetts Dartmouth, USA)
Strong stability preserving time-evolution of hyperbolic PDEs: bounds and barriers
Tuesday, May 26: 9:00 - 9:50, Location: M17.01 (KII)

Katrin Grunert (NTNU - Norwegian University of Science and Technology, Norway)
Numerical solutions of the nonlinear variational wave equation
Thursday, May 28: 9:55 - 10:35, Location: M17.02 (KII)

Michael Herty (RWTH Aachen, Germany)
Numerical Discretization of Conservation Laws Using Young Measures
Wednesday, May 27: 9:55 - 10:35, Location: M17.01 (KII)

Lorenzo Pareschi (Heriot Watt University, United Kingdom)
Structure-preserving neural surrogates for uncertainty quantification in kinetic equations
Monday, May 25: 11:20 - 12:00, Location: M17.01 (KII)

Gabriella Puppo (Università La Sapienza, Roma, Italy)
Preserving asymptotic limits at the discrete level
Wednesday, May 27:11:10 - 12:00, Location: M17.01 (KII)

Bangwei She (Capital Normal University, China)
A Lax-type framework for fluid models: convergence and error estimates
Tuesday, May 26: 9:55 - 10:35, Location: M17.01 (KII)

Vlad Vicol (New York University, USA)
Implosions in compressible Euler
Monday, May 25: 10:00 - 10:50, Location: M17.01 (KII)

Li Wang (University of Minnesota, USA)
Learn to evolve: self-supervised neural operator for nonlinear gradient flows
Tuesday, May 26: 9:55 - 10:35, Location: M17.02 (KII)

Kailiang Wu (Southern University of Science and Technology, Shenzhen, China)
Geometric Quasi-Linearization for Bound-Preserving Schemes: From General Framework to Unveiling the Hidden Algebraic-Differential Relation in MHD
Tuesday, May 26: 9:55 - 10:35, Location: M17.02 (KII)

Daily Program

Date: Monday, 25/May/2026	
8:30am	**Registration**
9:45am - 10:00am	**Opening** Location: **M17.01**
10:00am - 10:50am	**PL-01: Plenary Lecture** Location: **M17.01**
	Implosions in compressible Euler **Vlad Vicol**
10:50am - 11:20am	**Coffee Break**
11:20am - 12:00pm	**IL-01: Invited Lecture** Location: **M17.01**
	11:20am - 12:00pm **Structure-preserving neural surrogates for uncertainty quantification in kinetic equations** **Lorenzo Pareschi**
12:00pm - 2:00pm	**Lunch Break**
2:00pm - 3:40pm	**P1-1: Well-posedness and Regularity** Location: **M17.12 – 1.005**
	2:00pm - 2:25pm **Variational formulation of entropic solutions to nonlinear conservation laws** **Eitan Tadmor**
	2:25pm - 2:50pm **Temperature-driven turbulence in compressible fluid flows** **Yuhuan Yuan**
	2:50pm - 3:15pm **The energy-variational framework for binormal curvature flow** **Thomas Eiter, Robert Lasarzik, Marcel Śliwiński**
	3:15pm - 3:40pm **A Liouville-type theorem for 2x2 systems of hyperbolic conservation laws and applications** **Luca Talamini, Fabio Ancona, Elio Marcona**

2:00pm - 3:40pm	**P10-1: Active Flux/Central-Upwind** Location: **M17.02**
	2:00pm - 2:25pm **Recent development on active flux methodology** **Remi Abgrall, Yongle Liu, Walter Boscheri** **2:25pm - 2:50pm** **A finite element view on arbitrarily high-order accurate active flux methods** **Wasilij Barsukow** **2:50pm - 3:15pm** **Comparing the computational complexity of active flux and discontinuous Galerkin methods** **Wasilij Barsukow, Christian Klingenberg, Simon Krotsch** **3:15pm - 3:40pm** **Active flux method for hyperbolic problems using bicharacteristic evolution operators and WENO reconstruction** **Zhuyan Tang**
2:00pm - 3:40pm	**P14-1: Riemann Problems and Wave Patterns** Location: **M17.81 – 8.019**
	2:00pm - 2:25pm **A convergent front tracking scheme with an application to gas dynamics** **Manas Bhatnagar, Robin Young (lecture will be held by Robin Young)** **2:25pm - 2:50pm** **An exact Riemann solver for two layer two-phase flow model with Chaplygin pressure law** **Sheetal Prasad Thawait** **2:50pm - 3:15pm** **Exact Riemann solver for a laminar two-phase flow model with two velocities** **Raja Sekhar Tungala** **3:15pm - 3:40pm** **A Riemann problem in two-phase real fluid mixture** **Pardeep Kumar, Patricio Rosen**

2:00pm - 3:40pm	**P2-1: General Numerical Methods** Location: **M.17.17 – 1.017**
	2:00pm - 2:25pm **In-cell discontinuous reconstruction path-conservative methods for conservative systems in primitive variables** **Ernesto Pimentel-García, Mario Ricchiuto, Carlos Parés** **2:25pm - 2:50pm** **An energy preserving domain of dependence stabilization method on cut cell meshes** **Gunnar Birke, Christian Engwer, Sandra May, Louis Petri, Hendrik Ranocha** **2:50pm - 3:15pm** **Domain-of-dependence-stabilized cut-cell discretizations of linear kinetic models with summation-by-parts properties** **Louis Petri, Sigrun Ortleb, Gunnar Birke, Christian Engwer, Hendrik Ranocha** **3:15pm - 3:40pm** **Coupling a Maxey-Riley solver with Eulerian shallow-water solvers: contrasting finite-volume and E-SPH approaches.** **Rui M. L. Ferreira, Ibrahim Rahou, Ana M. Ricardo**
2:00pm - 3:40pm	**P3-1: Blowup and Stability** Location: **M.17.21 – 2.005**
	2:00pm - 2:25pm **Dynamic stability of Nash equilibria for a conservation law model of traffic flow** **Daniel Harmon, Alberto Bressan, Elio Marconi** **2:25pm - 2:50pm** **Analytical and numerical construction of tangent vectors to multi-dimensional scalar conservation laws** **Olivia Dreßen, Michael Herty, Adrian Kolb, Siegfried Müller** **2:50pm - 3:15pm** **Gas flow of mixtures on networks: existence of steady states with non-constant compressibility factors** **Alena Ulke, Michael Schuster, Simone Göttlich** **3:15pm - 3:40pm** **Time periodicity of global classical solution to radially symmetric unsteady flows with nonzero angular velocity in an annulus** **Jianli Liu**

2:00pm - 3:40pm	**P5-1: Singular Limits and Large Time Behaviour** Location: **M.17.23 – 2.011**
	2:00pm - 2:25pm **The zero capillarity limit for the Euler-Korteweg system with no-flux boundary conditions** **Paolo Antonelli, Yuri Cacchiò** **2:25pm - 2:50pm** **Large bulk viscosity limit for compressible MHD equations in critical Besov spaces** **Gennaro Ciampa, Donatella Donatelli, Giada Pellecchia** **2:50pm - 3:15pm** **Vanishing viscosity limit for strictly hyperbolic system of conservation laws in 1-d with nonlinear viscosity** **Boris Haspot, Animesh Jana** **3:15pm - 3:40pm** **Incompressible limits at large Mach number for a reduced compressible MHD system** **Aneta Wróblewska-Kamińska**
2:00pm - 3:40pm	**P6-1: Structure Preserving Methods** Location: **M.17.25 – 2.017**
	2:00pm - 2:25pm **On the heat GLM system and its compatible discretization** **Laura del Rio Martin, Firas Dhaouadi, Michael Dumbser** **2:25pm - 2:50pm** **Construction of asymptotic preserving schemes for kinetic models using upwind summation-by-parts operators** **Salim Elghawi, Sigrun Ortleb** **2:50pm - 3:15pm** **A second-order asymptotic-preserving IMEX scheme for the isentropic compressible Cahn-Hilliard-Navier-Stokes equations** **Andreu Martorell, Pep Mulet, Dionisio F. Yáñez** **3:15pm - 3:40pm** **A new asymptotic-preserving dual formulation finite-volume method for the compressible Euler equations** **Alina Chertock**

2:00pm - 3:40pm	**P7-1: Kinetic and Moment Equations** Location: **M17.74 – 7.017**
	2:00pm - 2:25pm **A semi-Lagrangian method for the polyatomic ESBGK model** **Klaas Willems, Erik Arlemark, Giovanni Samaey, Axel Klar** **2:25pm - 2:50pm** **Optimal convergence estimate of the limit from inverse power potential to hard sphere Boltzmann equation** **Zheng-Nan Hu, Jin Woo Jang, Zheng-An Yao, Yu-Long Zhou** **2:50pm - 3:15pm** **Build, analyse and simulate hierarchical PDEs for free-surface flows with Zoomy** **Ingo Steldermann, Julia Kowalski** **3:15pm - 3:40pm** **A model-adaptive discontinuous Galerkin scheme for linearized hierarchical moment equations** **Matthias Geratz, Manuel Torrilhon**
2:00pm - 3:40pm	**P8-1: Traffic Flow** Location: **M17.52 – 5.019**
	2:00pm - 2:25pm **Nonlocal models with time delay for mixed autonomous/human-driven traffic** **Ilaria Ciaramaglia, Paola Goatin, Gabriella Puppo** **2:25pm - 2:50pm** **Conservation laws with discontinuous gradient-dependent flux** **Debora Amadori, Alberto Bressan, Wen Shen** **2:50pm - 3:15pm** **A two-fluxes stochastic model of traffic waves** **Alberto Bressan, Sumantha Kanale Suresha** **3:15pm - 3:40pm** **Adapting priority Riemann solver for GSOM on road networks** **Caterina Balzotti, Roberta Bianchini, Maya Briani, Benedetto Piccoli**

2:00pm - 3:40pm	**P9-1: Higher Order Space Discretizations** Location: **M17.92 – 9.019**
	2:00pm - 2:25pm **An entropy stable discontinuous Galerkin spectral element method on heterogeneous grids** **Jens Keim, Anna Schwarz, Patrick Kopper, Christian Rohde, Andrea Beck** **2:25pm - 2:50pm** **Beyond polynomials: SBP operators for general function spaces** **Jan Glaubitz** **2:50pm - 3:15pm** **Superconvergence through SIAC filtering for discontinuous Galerkin solutions over non-uniform, structured meshes** **Roman Stuhlmacher, Jennifer Ryan** **3:15pm - 3:40pm** **Beyond polynomials: entropy stable and conservative discontinuous Galerkin methods with non-polynomial function spaces** **Henry Morten Haase, Jan Glaubitz, Andrew Winters**
3:40pm - 4:00pm	**Coffee Break**

4:00pm - 6:05pm	**P1-2: Well-Posedness and Regularity** Location: **M17.12 – 1.005**
	4:00pm - 4:25pm **Global well-posedness and asymptotic behavior of large strong solutions to viscous compressible fluids with variable coefficients** **Yachun Li**
	4:25pm - 4:50pm **Global well-posedness for the degenerate compressible Navier-Stokes equations with large data of spherical symmetry** **Gui-Qiang Chen, Jiawen Zhang, Shengguo Zhu**
	4:50pm - 5:15pm **Global existence and nonlinear stability of finite-energy solutions of the compressible Euler-Riesz equations with large initial data of spherical symmetry** **Carrillo José, Samuel Charles, Gui-Qiang Chen, Difan Yuan**
	5:15pm - 5:40pm **On weak solutions to self-organized systems of Euler-type with non-constant interaction kernel** **Gianmarco Cipollone, Debora Amadori, Cleopatra Christoforou**
	5:40pm - 6:05pm **Well posedeness for a tropical climate phase transition model** **Giada Cianfarani Carnevale, Donatella Donatelli, Stefano Spirito**

4:00pm - 6:05pm	P10-2: Active Flux/Central-Upwind Location: **M17.02**
	4:00pm - 4:25pm **An active flux method for compressible Navier-Stokes equations** **Junming Duan, Wasilij Barsukow, Klingenberg Christian** **4:25pm - 4:50pm** **A locally divergence-free local characteristic decomposition based path-conservative central-upwind scheme for ideal magnetohydrodynamics** **Shaoshuai Chu, Alexander Kurganov, Mária Lukácová-Medvidová, Mingye Na** **4:50pm - 5:15pm** **A low-dissipation central-upwind schemes for ideal MHD** **Yu-Chen Cheng, Praveen Chandrashekar, Christian Klingenberg** **5:15pm - 5:40pm** **Positivity-preserving new low-dissipation central-upwind schemes for compressible Euler equations** **Ruixiao Xin** **5:40pm - 6:05pm** **Bound- and positivity-preserving path-conservative central-upwind AWENO scheme for the five-equation model of compressible two-component flows** **Qingcheng Fu**

4:00pm - 6:05pm	**P12-1: Nonlocal Models** Location: **M17.81 – 8.019**
	4:00pm - 4:25pm **A nonlocal transport model for human blood cell dynamics** **Jan Friedrich, Wenhui Shi, Thomas Stiehl** **4:25pm - 4:50pm** **A nonlocal toy model for quasigeostrophic thermal active scalar** **Eduardo Abreu, John Pérez, Nelson Quispe-Cuba, Julio Valencia-Guevara** **4:50pm - 5:15pm** **Encryption and decryption through general non local balance laws** **Debora Amadori, Rinaldo M. Colombo, Andrea Salvadori** **5:15pm - 5:40pm** **Nonlocal regularizations of the Burgers equation** **Adrian Viorel** **5:40pm - 6:05pm** **Kermack-McKendrick type models for epidemics with nonlocal aggregation terms** **Fatemeh Ghaderi Zefreh**

4:00pm - 6:05pm	P2-2: General Numerical Methods Location: **M.17.17 – 1.017**
	4:00pm - 4:25pm **Realizability-preserving monolithic convex limiting in continuous Galerkin discretizations of the M_1 model of radiative transfer** **Paul Moujaes, Dmitri Kuzmin, Christian Bäumer**
	4:25pm - 4:50pm **A characteristic-based numerical scheme for the optimal control of production-destruction dynamics** **Simone Cacace, Alessio Oliviero, Mario Pezzella**
	4:50pm - 5:15pm **A multigrid-solved IMEX-AP scheme for the low-Froude shallow water equations** **Daniel Inzunza, Vincent Perrier**
	5:15pm - 5:40pm **Semi-implicit central scheme for hyperbolic systems of balance laws with relaxed source term** **Sudipta Sahu, Emanuele Macca, Rathan Samala**
	5:40pm - 6:05pm **Convergence of the non-staggered Nessyahu-Tadmor scheme for coupled systems of one-dimensional nonlocal balance laws** **Sanjibanee Sudha, Jan Friedrich, Rathan Samala**

4:00pm - 6:05pm	**P4-1: Probabilistic Models** Location: **M.17.21 – 2.005**
	4:00pm - 4:25pm **The zero-noise limit for hyperbolic conservation laws** **Ulrik Skre Fjordholm, Ola Mæhlen, Magnus Ch. Ørke** **4:25pm - 4:50pm** **On the dynamics of the 1D stochastic Quantum-Navier-Stokes equations** **Lorenzo Pescatore, Donatella Donatelli, Stefano Spirito** **4:50pm - 5:15pm** **Probabilistically strong solutions to stochastic Euler equations** **Benjamin Gess, Robert Lasarzik** **5:15pm - 5:40pm** **Mixing at the Batchelor scale for white-in-time flows** **Dennis Chemnitz, Robin Chemnitz** **5:40pm - 6:05pm** **Structure-preserving local discontinuous Galerkin methods for SPDEs with gradient noise** **Thomas Christiansen, Kenneth H. Karlsen**

4:00pm - 6:05pm	**P5-2: Singular Limits and Large Time Behaviour** Location: **M.17.23 – 2.011**
	4:00pm - 4:25pm **Asymptotic approximations of solutions to conservation laws near singularities.** **Alberto Bressan** **4:25pm - 4:50pm** **On the vanishing viscosity limit for inhomogeneous incompressible Navier-Stokes equations on bounded domains** **Jens Schröder, Emil Wiedemann** **4:50pm - 5:15pm** **Gradient regularization in viscoelasticity: analysis and asymptotic limits** **Aseel AlNajjar, Stefano Spirito, Athanasios Tzavaras** **5:15pm - 5:40pm** **Quasi-neutral limit and the mixer layer problem of Planck-Nernst-Poisson-Navier-Stokes equations for electro-hydrodynamics** **Shu Wang**

4:00pm - 6:05pm	**P6-2: Structure Preserving Methods** Location: **M.17.25 – 2.017**
	4:00pm - 4:25pm **Asymptotic-preserving discontinuous Galerkin method for the Navier Stokes equations with low Mach number** **Megala Anandan, Mária Lukáčová-Medvid'ová, Jiahui Zhang** **4:25pm - 4:50pm** **An asymptotic-preserving scheme for the low-Mach-number Euler equations via dissipative measure-valued solutions** **Arun Koottungal Revi, Amogh Krishnamurthy, Maria Lukacova-Medvidova** **4:50pm - 5:15pm** **Asymptotically compatible entropy consistent schemes for nonlocal conservation laws** **Nicola De Nitti, Kuang Huang** **5:15pm - 5:40pm** **An all-Froude semi-implicit solver for two-dimensional shallow-water** **José Ignacio Ramírez Fuentes, María de la Luz Muñoz Ruiz, Cipriano Escalante Sánchez, Tomás Morales de Luna** **5:40pm - 6:05pm** **Invariant region preserving schemes for mixed hyperbolic-parabolic SPDEs** **Thomas Christiansen, Kenneth H. Karlsen, Kristian Nekstad Maroni, Süleyman Ulusoy**

4:00pm - 6:05pm	**P7-2: Kinetic and Moment Equations** Location: **M17.74 – 7.017**
	4:00pm - 4:25pm **A moment based extension to the shallow water equations** **Robin Paar, Manuel Torrilhon** **4:25pm - 4:50pm** **Moment approximations to magnetic rotating shallow flows** **Michael Redle, Julian Koellermeier, Manuel Torrilhon** **4:50pm - 5:15pm** **Beyond the log law: Legendre polynomial velocity profiles for shear stress and sediment transport in shallow water models** **Afroja Parvin, Julian Koellermeier, Giovanni Samaey** **5:15pm - 5:40pm** **A hat function-based positive contact discontinuity capturing Boltzmann scheme** **Balwinder Singh, S. V. Raghurama Rao**

4:00pm - 6:05pm	**P8-2: Traffic Flow** Location: **M17.52 – 5.019**
	4:00pm - 4:25pm **A nonlocal upstream-downstream traffic flow model** **Luis-Miguel Villada-osorio** **4:25pm - 4:50pm** **Micro to macro: from infinite follow the leader ODEs to general traffic models** **Francesca Marcellini, Rinaldo M. Colombo, Graziano Guerra** **4:50pm - 5:15pm** **Nonlocal conservation laws with p-norm, the singular limit problem and applications to traffic flow** **Felisia Angela Chiarello, Alexander Keimer, Lukas Pflug** **5:15pm - 5:40pm** **Travelling waves and well-posedness in periodic nonlocal-local traffic models** **Rinaldo M. Colombo, Mauro Garavello, Claudia Nocita** **5:40pm - 6:05pm** **Conservation laws with coinciding smooth solutions in fluid dynamics and traffic flow** **Graziano Guerra, Rinaldo M. Colombo, Francesca Marcellini**

4:00pm - 6:05pm	**P9-2: Higher Order Space Discretizations** Location: **M17.92 – 9.019**
	4:00pm - 4:25pm **Entropy-stable high-order shock capturing on heterogeneous grids for discontinuous Galerkin spectral element methods** **Anna Schwarz, Jens Keim, Andrea Beck** **4:25pm - 4:50pm** **Nonlinearly stable open boundary conditions for shallow water flows** **Andrew Winters** **4:50pm - 5:15pm** **The high order unfitted discontinuous Galerkin methods for hyperbolic conservation laws in high dimension** **Pei Fu, Gunilla Kreiss, Sara Zahedi, Zelin Xin** **5:15pm - 5:40pm** **Stabilized P2-DG method with artificial viscosity for steady hyperbolic conservation laws** **Tiegang Liu** **5:40pm - 6:05pm** **A dispersive generalized Riemann problem for Galerkin methods** **Michel Tirapelle, Jens Keim, Christian Rohde**
6:05pm - 8:00pm	**Welcome Reception**

Date: Tuesday, 26/May/2026	
8:30am	**Registration**
9:00am - 9:50am	**PL-02: Plenary Lecture** Location: **M17.01**
	Strong stability preserving time-evolution of hyperbolic PDEs: bounds and barriers **Sigal Gottlieb**
9:55am - 10:35am	**IL-02: Invited Lecture** Location: **M17.01**
	9:55am - 10:35am **A Lax-type framework for fluid models: convergence and error estimates** **Bangwei She**
9:55am - 10:35am	**IL-03: Invited Lecture** Location: **M17.02**
	9:55am - 10:35am **Learn to evolve: self-supervised neural operator for nonlinear gradient flows** **Li Wang**
10:35am - 11:20am	**Coffee Break with Conference Photo**
11:20am - 12:00pm	**IL-04: Invited Lecture** Location: **M17.01**
	11:20am - 12:00am **Geometric Quasi-Linearization for bound-preserving schemes: From general framework to unveiling the hidden algebraic-differential relation in MHD** **Kailiang WU**
11:20am - 12:00pm	**IL-05: Invited Lecture** Location: **M17.02**
	11:20am - 12:00am **Two-dimensional Euler flows beyond Yudovich** **Elia Brue**
12:00pm - 2:00pm	**Lunch Break**

2:00pm - 3:40pm	**P1-3: Well-Posedness and Regularity** Location: **M17.12 – 1.005**
	2:00pm - 2:25pm **Regularity of velocity averages in kinetic equations with heterogeneity** **Kenneth Karlsen** **2:25pm - 2:50pm** **On the convergence of quasilinear viscous approximations using compensated compactness and kinetic formulation** **Ramesh Mondal** **2:50pm - 3:15pm** **Effective equations for a compressible liquid-vapor flow model** **Florian Wendt, Christian Rohde** **3:15pm - 3:40pm** **Hypersonic flow and Radon measure-valued solution** **Aifang QU**
2:00pm - 3:40pm	**P12-2: Nonlocal Models** Location: **M17.02**
	2:00pm - 2:25pm **Singular limit for a class of nonlocal conservation laws via compensated compactness** **Giuseppe Maria Coclite, Nicola De Nitti, Kuang Huang** **2:25pm - 2:50pm** **Monotone-based numerical schemes for two-dimensional systems of nonlocal conservation laws** **Anika Beckers, Jan Friedrich, Michael Herty** **2:50pm - 3:15pm** **On the existence and uniqueness of nonlocal nonlinear conservation laws by means of fixed-point methods** **Hossein Nick Zinat Matin, Alexander Keimer, Lorenzo Liverani** **3:15pm - 3:40pm** **Measure solutions, smoothing effect, and deterministic particle approximation for a conservation law with non-local flux** **Marco Di Francesco, Simone Fagioli, Emanuela Radici**

2:00pm - 3:40pm	**P13-1: Multiscale and Homogenization** Location: **M.17.17 – 1.017**
	2:00pm - 2:25pm **Multiscale modelling of cancer invasion and metastasis: the effects of tumour heterogeneity** **Dimitrios Katsaounis, Nikolaos Sfakianakis, Mark AJ Chaplain** **2:25pm - 2:50pm** **Multiscale hyperbolic-parabolic models for nonlinear reactive transport in heterogeneously fractured porous media** **Taras Melnyk, Sorin Pop, Christian Rohde** **2:50pm - 3:15pm** **Numerical homogenization of multiscale hyperbolic conservation laws** **Giulia Merlini, Claude Le Bris, Frederic Legoll** **3:15pm - 3:40pm** **Modeling edema dynamics in soft tissue using mixture theory** **Simone Bonfiglio, Ana Carpio**
2:00pm - 3:40pm	**P14-2: Riemann Problems and Wave Patterns** Location: **M17.81 – 8.019**
	2:00pm - 2:25pm **Do Riemann solutions consist of rarefactions, shocks, and constants?** **Bradley J. Plohr, Stephen Schecter, Dan Marchesin** **2:25pm - 2:50pm** **Viscosity-dependent wave patterns in the Riemann problem for nonstrictly hyperbolic conservation laws** **Valentin Pellhammer** **2:50pm - 3:15pm** **Maximal energy dissipation principle and the Riemann problem for shallow water flow** **Sanja Ružičić**

2:00pm - 3:40pm	**P15-1: Numerical Analysis** Location: **M17.92 – 9.019**
	2:00pm - 2:25pm **A posteriori error estimates for high order approximations of hyperbolic balance laws** **Aleksey Sikstel, Jan Giesselmann, Graziano Guerra** **2:25pm - 2:50pm** **A posteriori error analysis of a Runge-Kutta discontinuous Galerkin scheme for nonlinear convection-diffusion systems in convection-dominated flows** **Andreas Dedner, Jan Giesselmann, Kiwoong Kwon, Tristian Pryer** **2:50pm - 3:15pm** **Error estimates of an asymptotic-preserving discontinuous Galerkin method for the continuous data assimilation algorithm of Navier-Stokes system** **Megala Anandan, Mária Lukáčová-Medvid'ová, Jiahui Zhang** **3:15pm - 3:40pm** **Numerical methods for the Camassa-Holm equation in the zero-filter limit** **Dimitrios Geladaris, Christian Rohde**
2:00pm - 3:40pm	**P16-1: Dispersive Waves** Location: **M17.52 – 5.019**
	2:00pm - 2:25pm **Dispersive waves in quasilinear hyperbolic systems on a periodic background** **Giovanni Russo** **2:25pm - 2:50pm** **Modeling non-propagating modes by dispersive hyperbolic systems** **Ilya Peshkov, Loïc Le Marrec** **2:50pm - 3:15pm** **Dispersive boundary layers for scalar conservation laws in one space dimension and their stability** **Paolo Antonelli, Pierangelo Marcati, Laura V. Spinolo** **3:15pm - 3:40pm** **Effective dispersive model and resonance in fluids with periodic geometry** **Laila Busaleh, David Ketcheson**

2:00pm - 3:40pm	**P3-2: Blowup and Stability** Location: **M.17.21 – 2.005**
	2:00pm - 2:25pm **Repeated blowup for Navier-Stokes solutions** **James Glimm** **2:25pm - 2:50pm** **Stability of planar rarefaction waves in the vanishing dissipation limit of the Navier–Stokes–Fourier system** **Wladimir Neves, Eduard Feireisl** **2:50pm - 3:15pm** **Density of "wild" initial data for the isentropic Euler equations considering global-in-time admissible weak solutions** **Simon Markfelder, Daniel Boutros** **3:15pm - 3:40pm** **Weak-BV stability for Euler with boundaries** **Jiayun Meng, Alexis Vasseur, Moon-Jin Kang, HyeonSeop Oh**
2:00pm - 3:40pm	**P5-3: Singular Limits and Large Time Behaviour** Location: **M.17.23 – 2.011**
	2:00pm - 2:25pm **Dissipative structure of higher order regularizations of hyperbolic systems of conservation laws in several space dimensions** **Felipe Angeles, Ramón G. Plaza, José Manuel Valdovinos** **2:25pm - 2:50pm** **Formal derivation of a stratified compressible two-phase flow model** **Pierrick Le Vourc'h** **2:50pm - 3:15pm** **Hyperbolic regularization of free-surface flow models** **Julian Koellermeier** **3:15pm - 3:40pm** **The Cattaneo-Christov system for compressible fluid flow in several space dimensions: Well-posedness and non-hyperbolicity** **Felipe Angeles**

2:00pm - 3:40pm	**P6-3: Structure Preserving Methods** Location: **M.17.25 – 2.017**
	2:00pm - 2:25pm **A simple and general framework for the construction of exactly div-curl-grad compatible discontinuous Galerkin finite element schemes on unstructured simplex meshes** **Michael Dumbser, Remi Abgrall, Pierre-Henri Maire** **2:25pm - 2:50pm** **A thermodynamically compatible scheme for continuum mechanics with torsion** **Alessia Lucca, Ilya Peshkov, Michael Dumbser** **2:50pm - 3:15pm** **A Hamiltonian approach to entropy-stable and well-balanced schemes** **Patrick Ersing, Andrew R. Winters** **3:15pm - 3:40pm** **How to make fully discrete entropy stability of three-point well-balanced finite-volume schemes** **Christophe Berthon, Manuel Castro Díaz, Ludovic Martaud, Tomás Morales de Luna**
2:00pm - 3:40pm	**P7-3: Kinetic and Moment Equations** Location: **M17.74 – 7.017**
	2:00pm - 2:25pm **H-theorems for dense reactive mixtures and global existence of solutions** **Jacek Polewczak** **2:25pm - 2:50pm** **Gramian moment closures for kinetic equations: hyperbolicity and numerics** **Eda Yilmaz, Manuel Torrilhon** **2:50pm - 3:15pm** **Consistent kinetic models for quantum gases - modeling and long-time behavior** **Marlies Pirner** **3:15pm - 3:40pm** **Equilibrium boundary conditions for vectorial multi-dimensional lattice Boltzmann schemes** **Thomas Bellotti, Denise Aregba-Driollet**
3:40pm - 4:00pm	**Coffee Break**

4:00pm - 6:05pm	**P1-4: Well-Posedness and Regularity** Location: **M17.12 – 1.005**
	4:00pm - 4:25pm **Nonlinear sound waves in compressible Euler** **Robin Young** **4:25pm - 4:50pm** **On Lions density patch at the critical level** **Alessandro Violini, Stefan Škondrić** **4:50pm - 5:15pm** **Global small solutions to the 2D non-resistive compressible MHD system near an equilibrium** **Yi Zhu** **5:15pm - 5:40pm** **Global existence/non-existence of classical solution for isentropic Euler equations with time dependent damping** **Balakrishna Chhatria** **5:40pm - 6:05pm** **Well-posedness and numerical approximation of nonlinear conservation laws with hysteresis** **Fabio Bagagiolo, Paola Goatin, Stefan Moreti**
4:00pm - 6:05pm	**P14-3: Riemann Problems and Wave Patterns** Location: **M17.81 – 8.019**
	4:00pm - 4:25pm **Formation of vacuum state and delta shock in the solution of two-dimensional Riemann problem for zero pressure gas dynamics** **Anamika Pandey** **4:25pm - 4:50pm** **Generalised Riemann problem and wave interactions in a macroscopic production model** **Lilu Sahu** **4:50pm - 5:15pm** **Dynamics of delta shock waves originating from a three pieces Riemann problem for traffic flow model** **Priyanka, Dr. M. Zafar**

4:00pm - 6:05pm	**P15-2: Numerical Analysis** Location: **M17.92 – 9.019**
	4:00pm - 4:25pm **Convergence of a finite volume scheme for the Navier-Stokes-Korteweg equations to measure-valued solutions** **Jan Giesselmann, Philipp Öffner, Robert Sauerborn** **4:25pm - 4:50pm** **Convergence of a discretized observer for the Euler equations** **Aidan Chaumet, Jan Giesselmann** **4:50pm - 5:15pm** **Dissipative weak solutions for the two-component Euler-system** **Jaya Agnihotri, Philipp Öffner** **5:15pm - 5:40pm** **Numerical study of dissipative measure-valued solutions for the compressible Euler system** **Changsheng Yu** **5:40pm - 6:05pm** **Numerical error estimates for the compressible Navier-Stokes equations with potential temperature transport** **Mária Lukáčová-Medvid'ová, Andreas Schömer**

4:00pm - 6:05pm	**P17-1: Dimensionally Reduced Flows** Location: **M17.52 – 5.019**
	4:00pm - 4:25pm **A double coordinate system for submarine landslides and generated tsunamis** **Gwendal Léger, François Bouchut, Enrique Domingo Frenández-Nieto, Anne Mangeney, Gladys Narbona-Reina** --- **4:25pm - 4:50pm** **A depth-integrated model for dry granular flows in local curvilinear coordinates with friction correction** **Sergio Valiente Ávila, Tomás Morales de Luna, José Garres Díaz, Manuel Jesús Castro Díaz** --- **4:50pm - 5:15pm** **A new model for two-layer liquid-gas stratified flows in pipes with general cross sections** **Gerardo Hernández-Dueñas, Sarswati Shah** --- **5:15pm - 5:40pm** **Hyperbolic modelling for thin film flows** **Rahul Barthwal, Christian Rohde, Yue Wang** --- **5:40pm - 6:05pm** **Conservative discretization of a nonlinear hyperbolic phase-transport system with curvature** **Baskar Sambandam, Veerappa Gowda**

4:00pm - 6:05pm	**P2-3: General Numerical Methods** Location: **M.17.17 – 1.017**
	4:00pm - 4:25pm **Efficient WENO schemes for nonuniform grids with optimal accuracy** **Mari Carmen Martí, Pep Mulet, Dionisio F. Yáñez, David Zorío** **4:25pm - 4:50pm** **Interpretation of distributed residual methods with piece-wise defined test functions** **Pierre-Marie Boulvard** **4:50pm - 5:15pm** **Numerical simulations of single-mode Rayleigh--Taylor and Richtmyer--Meshkov instabilities with improved front-tracking/ghost-fluid method** **Tulin Kaman** **5:15pm - 5:40pm** **Numerical investigation of nonlinear age-structured tumor cell population model** **Paramjeet Singh**
4:00pm - 6:05pm	**P3-3: Blowup and Stability** Location: **M.17.21 – 2.005**
	4:00pm - 4:25pm **Development of implosions of solutions of the three-dimensional degenerate compressible Navier-Stokes equations** **Gui-Qiang G Chen, Lihui Liu, Shengguo Zhu** **4:25pm - 4:50pm** **The estimate of lifespan for ideal incompressible MHD systems** **Mengni Li, Hongwei Yuan** **4:50pm - 5:15pm** **Stability analysis for the generalized viscous Burgers' equation with time delay** **Yoshihiro Ueda** **5:15pm - 5:40pm** **Similarity solutions and nonlinear wave interaction in a new hyperbolic model of two-phase thin film flow of perfectly soluble antisurfactant** **Hari Om Jangid, T Raja Sekhar** **5:40pm - 6:05pm** **Asymptotic stability of steady transonic flows in nozzles** **Masashi Ohnawa**

4:00pm - 6:05pm	**P4-2: Probabilistic Models** Location: **M17.02**
	4:00pm - 4:25pm **Uncertainty quantification of conservation laws with several random parameters** **Juliette Dubois, Michael Herty, Siegfried Müller** **4:25pm - 4:50pm** **Tensor train discontinuous Galerkin with multiresolution-based grid adaptation for high-dimensional random hyperbolic conservation laws** **Juliette Dubois, Adrian Kolb, Michael Herty, Siegfried Müller, Tim A. Werthmann** **4:50pm - 5:15pm** **A multilevel strategy for high-dimensional random hyperbolic conservation laws** **Kai Hinzmann, Michael Herty, Adrian Kolb, Siegfried Müller** **5:15pm - 5:40pm** **Uncertainty quantification for nonlocal traffic flow using stochastic Galerkin approximations** **Fabian Ziegler, Simone Göttlich** **5:40pm - 6:05pm** **Stochastic nonlocal traffic flow modeling** **Timo Böhme, Simone Göttlich, Andreas Neuenkirch**
4:00pm - 6:05pm	**P5-4: Singular Limits and Large Time Behaviour** Location: **M.17.23 – 2.011**
	4:00pm - 4:25pm **L^2-contraction for viscous-dispersive shock waves of KdV-Burgers equation** **Geng Chen, Namhyun Eun, Moon-Jin Kang, Yannan Shen** **4:25pm - 4:50pm** **On the asymptotic behavior of irrotational steady Euler equations** **Tian-Yi Wang** **4:50pm - 5:15pm** **Long time behavior of stochastic isentropic Euler equation with linear damping** **Rongyi Dai, Jeffrey Kuan, Krutika Tawri, Suncica Canic, Konstantina Trivisa**

4:00pm - 6:05pm	**P6-4: Structure Preserving Methods** Location: **M.17.25 – 2.017**
	4:00pm - 4:25pm **Well-balanced semi-implicit relaxed finite volume schemes for hyperbolic balance laws** **Irene Gómez-Bueno, Manuel Jesús Castro Díaz, Andrea Thomann** **4:25pm - 4:50pm** **Development of discontinuous Galerkin methods for hyperbolic systems that preserve a divergence or a curl constraint** **Jonathan Jung, Vincent Perrier** **4:50pm - 5:15pm** **Numerical study of polydisperse sedimentation in complex geometries using an invariant-region-preserving scheme** **Juan Barajas-Calonge, Raimund Bürger, Pep Mulet, Luis Miguel Villada** **5:15pm - 5:40pm** **Entropy stable numerical schemes for divergence diminishing Chew, Goldberger & Low equations for plasma flows** **CHETAN SINGH, Harish Kumar, Deepak Bhoriya, Dinshaw S. Balsara** **5:40pm - 6:05pm** **Quantification of numerical entropy loss for ocean models with a projection method** **Nina Aguillon, Marie Boussard, Julien Salomon**
4:00pm - 6:05pm	**P7-4: Kinetic and Moment Equations** Location: **M17.74 – 7.017**
	4:00pm - 4:25pm **A kinetic framework for SIR-type epidemic models: mean-field limit and asymptotic analysis** **Giorgio Martalò** **4:25pm - 4:50pm** **Hydrodynamic limit of the Boltzmann equation towards Riemann solutions composed of shocks** **Mingi Choe, Moonjin Kang, Chanwoo Kim** **4:50pm - 5:15pm** **Long-time asymptotic analysis of the Vlasov equation under a strong magnetic field** **Manuela Trullo, Francis Filbet, Luis Miguel Rodrigues**

Date: Wednesday, 27/May/2026	
8:30am	**Registration**
9:00am - 9:50am	**PL-03: Plenary Lecture** Location: **M17.01**
	High-order well-balanced schemes for balance laws: recent advances, challenges, and numerical results **Manuel Jesus Castro Diaz**
9:55am - 10:35am	**IL-06: Invited Lecture** Location: **M17.01**
	9:55am - 10:35am **Numerical discretization of conservation laws using Young measures** **Michael Herty**
9:55am - 10:35am	**IL-07: Invited Lecture** Location: **M17.02**
	9:55am - 10:35am **Singular limits in some nonlinear continuity equations** **Gianluca Crippa**
10:35am - 11:10am	**Coffee Break**
11:10am - 12:00pm	**PL-04: Plenary Lecture** Location: **M17.01**
	Preserving asymptotic limits at the discrete level **Gabriella Puppo**
12:00pm - 2:00pm	**Lunch Break**
2:00pm - 6:05pm	**Free Discussions and Excursions**

Date: Thursday, 28/May/2026	
8:30am	**Registration**
9:00am - 9:50am	**PL-05: Plenary Lecture** Location: **M17.01**
	Structure and asymptotic preserving schemes for the Vlasov-Poisson system and related models **Francis Filbet**
9:55am - 10:35am	**IL-08: Invited Lecture** Location: **M17.01**
	9:55am - 10:35am **Recent advances in subcell based limiting for discontinuous Galerkin methods with guarantees** **Gregor Gassner**
9:55am - 10:35am	**IL-09: Invited Lecture** Location: **M17.02**
	9:55am - 10:35am **Numerical solutions of hyperbolic equations – wave breaking leads the way** **Katrin Grunert**
10:35am - 11:10am	**Coffee Break**
11:10am - 12:00pm	**P18-1: Scientific Computing** Location: **M17.01**
	11:10am - 11:35am **Adaptive schemes for hyperbolic systems of conservation laws** **Alexander Kurganov** --- **11:35am - 12:00pm** **Quantum computation of Hamilton-Jacobi equations: multivalued and viscosity solutions, and quantum scientific computing platform "UnitaryLab"** **Shi Jin**

11:10am - 12:00pm	**P19-1: Hyperbolic Modelling** Location: **M17.02**
	11:10am - 11:35am **Localization phenomena in chemotaxis and in inelastic fracture** **Athanasios Tzavaras** **11:35am - 12:00pm** **On the equivalence of some non-conservative hyperbolic systems and conservative systems** **Gerald Warnecke, Ferdinand Thein**
12:00pm - 2:00pm	**Lunch Break**
2:00pm - 3:40pm	**P11-1: Higher Order Time Discretizations** Location: **M17.12 – 1.005**
	QUINPI: high order implicit schemes for conservation laws **Gabriella Puppo, Matteo Semplice, Giuseppe Visconti, Michal Zeravy** **Extending Quinpi in 2D: an implicit scheme for nonlinear conservation laws** **Alessandra Zappa, Matteo Semplice** **Stability analysis of ADER-DG methods on classical and degenerate spacetime geometries** **Mauro Bonafini, Davide Torlo, Elena Gaburro** **ADER schemes go faster and faster** **Simone Chiocchetti, Gregor Gassner**

2:00pm - 3:40pm	**P14-4: Riemann Problems and Wave Patterns** Location: **M17.81 – 8.019**
	2:00pm - 2:25pm **On the characteristic fields of averaged models of two phase flows** **Michael Ndjinga** **2:25pm - 2:50pm** **Convergence to superposition of boundary layer, rarefaction and shock for the inflow problem of the 1D Navier-Stokes equations** **Sungho Han, Moon-Jin Kang, Jeongho Kim, Nayeon Kim, HyeonSeop Oh** **2:50pm - 3:15pm** **A local-in-time quadratic functional for scalar conservation laws** **Egor Baitenov, Stefano Modena** **CANCELLED** **Effects of quartic nonlinearity on shock wave propagation in non-ideal media** **Neelam Neelam**
2:00pm - 3:40pm	**P2-4: General Numerical Methods** Location: **M.17.17 – 1.017**
	2:00pm - 2:25pm **A IMEX-based spectral scheme with adaptive time-stepping for the Vlasov-Poisson system in the quasi-neutral limit** **Bernardo Collufio** **2:25pm - 2:50pm** **A finite volume characteristics scheme for compressible non-equilibrium multiphase flows** **Wassim Aboussi, Kevin Pons** **2:50pm - 3:15pm** **Barotropic-baroclinic splitting for multilayer shallow water models with exchanges** **Nina Aguillon, Sophie Hörnschemeyer, Jacques Sainte-Marie** **3:15pm - 3:40pm** **A provably convergent MUSCL-type central scheme for conservation laws with discontinuous flux** **Nikhil Manoj, Sudarshan Kumar K**

2:00pm - 3:40pm	**P20-1: Mixed-Type Problems** Location: **M.17.23 – 2.011**
	2:00pm - 2:25pm **Shock wavefronts for forward-backward-forward parabolic equations** **Diego Berti, Andrea Corli, Luisa Malaguti** **2:25pm - 2:50pm** **Many species system with nonlocal interactions driven by Riesz potentials** **Valeria Iorio** **2:50pm - 3:15pm** **Localization of self-similar solutions in diffusion–relaxation systems** **Hoyoun Kim**
2:00pm - 3:40pm	**P21-1: Relaxation Approximation** Location: **M.17.21 – 2.005**
	2:00pm - 2:25pm **Relaxation schemes for coupled systems of conservation laws** **Niklas Kolbe, Siegfried Müller, Aleksey Sikstel** **2:25pm - 2:50pm** **Hyperbolic relaxation of the Navier-Stokes-Cahn-Hilliard system for incompressible two-phase flows** **Hasel Cicek Konan, Christian Rohde** **2:50pm - 3:15pm** **A hyperbolic approximation of the nonlinear Schrödinger equation** **Abhijit Biswas, Laila, B. Busaleh, David, I. Ketcheson, Carlos Muñoz-Moncayo, Manvendra Rajvanshi** **3:15pm - 3:40pm** **Relative entropy method for relaxation of fluid systems** **Christina Mahmoud, Hélène Mathis, Nicolas Seguin**

2:00pm - 3:40pm	**P23-1: Control and Inverse Problems** Location: **M17.74 – 7.017**
	2:00pm - 2:25pm **Existence of spiral strategies for blocking fire spreading** **Stefano Bianchini, Martina Zizza** **2:25pm - 2:50pm** **On the optimal control of propagation fronts** **Chiara Trifone, Stefano Bianchini** **2:50pm - 3:15pm** **Direct and inverse problems in a hypersonic piston model with large velocity variations** **Dian Hu, Qianfeng Li, Yongqian Zhang**
2:00pm - 3:40pm	**P25-1: Machine Learning Methods** Location: **M17.02**
	2:00pm - 2:25pm **Stochastic collocation for the Euler system and neural networks** **Mária Lukácová-Medvid'ová, Simon Schneider** **2:25pm - 2:50pm** **Hybrid modeling of hyperbolic systems of equations and auto-differentiability** **Bruno Després** **2:50pm - 3:15pm** **Failsafe continuous Galerkin discretizations with stabilizations based on artificial intelligence** **Hennes Hajduk**

2:00pm - 3:40pm	**P26-1: Symmetry Methods** Location: **M17.52 – 5.019**
	2:00pm - 2:25pm **A unified symmetry framework for an Aw–Rascle model with Chaplygin gas dynamics** **Sandhya Maurya, Tapan Kumar Hota** **2:25pm - 2:50pm** **Fast reconnection for incompressible MHD equations** **Gennaro Ciampa** **2:50pm - 3:15pm** **Symmetry analysis of a transport-enhanced blood flow model** **Debendra Prasad Panda, Manoj Pandey** **3:15pm - 3:40pm** **Semi-analytical solution of $(2+1)$-dimensional KdV-Burgers equation using homotopy analysis method** **Rajan Arora, Prince Sharma**
2:00pm - 3:40pm	**P6-5: Structure Preserving Methods** Location: **M.17.25 – 2.017**
	2:00pm - 2:25pm **A second-order invariant-region-preserving scheme for a transport-flow model of polydisperse sedimentation** **Juan Barajas-Calonge, Raimund Bürger, Pep Mulet, Luis Miguel Villada** **2:25pm - 2:50pm** **A positivity-preserving segregated Runge-Kutta finite volume scheme for compressible flows on staggered meshes.** **Thomas Harbreteau, Raphaèle Herbin, Jean-Claude Latché** **2:50pm - 3:15pm** **Physical constraint preserving alternative finite-difference WENO scheme for hyperbolic systems with non-conservative products** **Deepak Bhoriya, Dinshaw S. Balsara, Praveen Chandrashekar, Chi-Wang Shu** **3:15pm - 3:40pm** **A kinetic energy preserving discontinuous Galerkin scheme based on a flexible velocity kinetic model** **Kedar Wagh, S.V. Raghurama Rao, Sashikumaar Ganesan**

2:00pm - 3:40pm	**P9-3: Higher Order Space Discretizations** Location: **M17.92 – 9.019**
	2:00pm - 2:25pm **A structure-preserving compact high-order method for multi-dimensional hyperbolic conservation laws** **Christian Klingenberg** **2:25pm - 2:50pm** **High-order flux splitting schemes for the Euler equations of gas dynamics** **Shaoshuai Chu, Michael Herty, Eleuterio F. Toro** **2:50pm - 3:15pm** **An adaptive spectral element method for systems of conservation laws** **Manuel Colera--Rico, Vít Dolejsí** **3:15pm - 3:40pm** **A high-order structure-preserving scheme for a model for two-layer thin film flow** **Jan Zawallich, Rahul Barthwal, Philipp Öffner**
3:40pm - 4:00pm	**Coffee Break**

4:00pm - 6:05pm	**P1-5: Well-Posedness and Regularity** Location: **M17.12 – 1.005**
	4:00pm - 4:25pm **Hölder continuity for multi-d balance laws** **Laura Caravenna, Alexander Cliffe, Elio Marconi, Andrea Pinamonti** --- **4:25pm - 4:50pm** **Regularity of Lagrangian flows associated to singular vector fields** **Henrique de Souza, João Fernando Nariyoshi** --- **4:50pm - 5:15pm** **Oleinik inequality for balance laws** **Rida Harb, Stéphane Junca** --- **5:15pm - 5:40pm** **On some typicality and density results for nonsmooth vector fields and the associated continuity equation** **Francesco Cianfrocca, Stefano Modena** --- **5:40pm - 6:05pm** **Supercritical geometric optics and expected regularity for scalar multi-dimensional nonlinear conservation laws** **Christophe Cheverry, Rida Harb, Stéphane Junca**

4:00pm - 6:05pm	**P14-5: Riemann Problems and Wave Patterns** Location: **M17.81 – 8.019**
	4:00pm - 4:25pm **Self-similar analysis of strong shock waves in a magnetized, non-ideal, self-gravitating gas with finite electrical conductivity** **Sanjay Yadav, Priyanshu Yadav** --- **4:25pm - 4:50pm** **Topological structures in systems of conservation laws** **César de Souza Eschenazi, Wanderson José Lambert, Edwin Pedro López Bambarén, Marlon Michael López-Flores, Dan Marchesin, Carlos Frederico Borges Palmeira** --- **4:50pm - 5:15pm** **Shock wave evolution and conservation laws in compressible flow via symmetry based analysis** **Prince Sharma, Rajan Arora** --- **5:15pm - 5:40pm** **Riemann problem for a pressureless gas dynamics system with time-gradually-degenerate linear damping** **Richard De la cruz, Wladimir Neves** --- **5:40pm - 6:05pm** **Interactions of delta shock waves in generalized Chaplygin gas with source terrm** **Minhajul Minhajul**

4:00pm - 6:05pm	**P2-5: General Numerical Methods** Location: **M.17.17 – 1.017**
	4:00pm - 4:25pm **Supersonic reacting jet flows from three-dimensional divergent conical nozzles** **Wancheng Sheng** **4:25pm - 4:50pm** **Experimental and non-hydrostatic Saint-Venant-Exner modeling of dike breach by overtopping** **Jorge Moya, Ricardo J. L Jónatas, Rui M. L Ferreira, Silvia Amaral, C. Escalante, Enrique D. Fernandez-Nieto** **4:50pm - 5:15pm** **Combined compressible-incompressible multiphase modelling approach of solid propellant ignition and combustion using an extended discontinous Galerkin method** **Daniel Tomaschewski** **5:15pm - 5:40pm** **Smooth perturbations of diagonally implicit Runge–Kutta methods** **Zachary Grant, John Driscoll, Sigal Gottlieb, César Herrera, Tej Sai Kakumanu, Monica Stephens**
4:00pm - 6:05pm	**P20-2: Mixed-Type Problems** Location: **M.17.23 – 2.011**
	4:00pm - 4:25pm **The Hele-Shaw free boundary limit of Buckley-Leverett System** **Wladimir Neves, André Gomes** **4:25pm - 4:50pm** **A bulk-surface Cahn-Hilliard model for chemically active wetting** **Megha Kamath Katapady** **4:50pm - 5:15pm** **String stability for hysteretic flows** **Haitao Fan, Andrea Corli**

4:00pm - 6:05pm	**P21-2: Relaxation Approximation** Location: **M.17.21 – 2.005**
	4:00pm - 4:25pm **On hyperbolic approximations for a class of dispersive and diffusive-dispersive equations** **Rahul Barthwal, Firas Dhaouadi, Christian Rohde** **4:25pm - 4:50pm** **A hyperbolic relaxation approximation of the incompressible Navier-Stokes equations with artificial compressibility** **Qian Huang, Christian Rohde, Wen-An Yong, Ruixi Zhang** **4:50pm - 5:15pm** **A structure preserving scheme for the Alber equation, and investigation of the onset of instability** **Agissilaos Athanassoulis, Irene Kyza, Fotini Karakatsani** **5:15pm - 5:40pm** **Well-balanced kinetic relaxation schemes for hyperbolic systems of balance laws** **León Miguel Ávila León, Manuel Jesús Castro Díaz, Victor Michel-Dansac, Emmanuel Frank**
4:00pm - 6:05pm	**P22-1: ALE Methods** Location: **M.17.25 – 2.017**
	4:00pm - 4:25pm **An arbitrary Lagrangian-Eulerian semi-implicit FV/FE method for continuum mechanics** **Saray Busto** **4:25pm - 4:50pm** **SCOUT: Semi-Lagrangian COnservative and Unconditionally sTable schemes for nonlinear advection-diffusion problems** **Silvia Preda, Walter Boscheri, Matteo Semplice, Maurizio Tavelli** **4:50pm - 5:15pm** **An Arbitrary-Lagrangian-Eulerian solver for relativistic detonation waves** **Sara Rinaldi, Olindo Zanotti, Michael Dumbser** **5:15pm - 5:40pm** **Simplex solver for Lagrangian hydrodynamics** **Xihua Xu, Min Xiao**

4:00pm - 6:05pm	**P23-2: Control and Inverse Problems** Location: **M17.74 – 7.017**
	4:00pm - 4:25pm **Galois connections in Hamilton - Jacobi equations and conservation laws** **Rinaldo M. Colombo, Vincent Perrollaz** **4:25pm - 4:50pm** **Discrete adjoint gradient computation for multiclass traffic flow models on road networks** **Carmen Mezquita-Nieto, Paola Goatin, Axel Klar** **4:50pm - 5:15pm** **On controllability of conservation laws with space discontinuous flux** **Fabio Ancona, Luca Talmin** **5:15pm - 5:40pm** **A robust optimization framework for sensor networks** **Vanshika Datta, Chandal Nahak** **5:40pm - 6:05pm** **High-fidelity adjoint aerodynamic optimization design based on high-order CFD solvers for NS equations** **Kun Wang, Tiegang Liu, Bin Zhang, Jiaxin Wang**

4:00pm - 6:05pm	**P24-1: Relativistic Flows** Location: **M17.52 – 5.019**
	4:00pm - 4:25pm **Relativistic Navier-Stokes models** **Heinrich Freistühler** **4:25pm - 4:50pm** **High order numerical discretizations of the Einstein-Euler equations in the generalized harmonic formulation** **Stefano Muzzolon, Michael Dumbser, Olindo Zanotti, Elena Gaburro** **4:50pm - 5:15pm** **The relativistic Euler equations with Synge energy** **Ferdinand Thein, Tommaso Ruggeri, Qinghua Xiao** **5:15pm - 5:40pm** **Constraints preserving Lax-Wendroff flux reconstruction for relativistic hydrodynamics with general equations of state** **Sujoy Basak, Arpit Babbar, Harish Kumar, Praveen Chandrashekar** **CANCELLED** **Provably physical-constraints-preserving alternative finite difference-WENO scheme for special relativistic hydrodynamic (RHD) equations** **Shubham Upadhyay, Biswarup Biswas, Rakesh Kumar**
4:00pm - 6:05pm	**P25-2: Machine Learning Methods** Location: **M17.02**
	4:00pm - 4:25pm **Machine learning–enhanced junction modeling for shallow-water channel** **Djahou Tognon, Magali Ribot, Bruno Galerne** **4:25pm - 4:50pm** **A structure-preserving numerical method based on a neural network for a hyperbolic relaxation system** **Yingjie Song, Christian Rohde** **4:50pm - 5:15pm** **A fully-learnable PDE-informed framework for conservation laws** **Eduardo Abreu, Joao Florindo, Wanderson Lambert**

4:00pm - 6:05pm	**P9-4: Higher Order Space Discretizations** Location: **M17.92 – 9.019**
	4:00pm - 4:25pm **Well-balanced high-order finite-difference methods based on a local flux formulation** **Carlos Parés, Maria Kazolea, Baifen Ren** **4:25pm - 4:50pm** **Entropy analysis and entropy stable DG methods for the shallow water moment equations** **Julio Careaga, Patrick Ersing, Julian Koellermeier, Andrew R. Winters** **4:50pm - 5:15pm** **OEDG: Oscillation-eliminating discontinuous Galerkin method for hyperbolic conservation laws** **Manting Peng, Zheng Sun, Kailiang Wu** **5:15pm - 5:40pm** **Oscillation-eliminating central DG schemes on Cartesian meshes for hyperbolic conservation laws** **Manting Peng, Kailiang Wu, Caiyou Yuan**
7:30pm	**Conference Dinner** Location: **Alte Reithalle**

Date: Friday, 29/May/2026	
9:00am - 9:50am	**IL-10: James Glimm Lecture** Location: **Neues Schloss – Weißer Saal**
	9:00am - 9:50am **From real to ideal compressible fluids** **Eduard Feireisl**
9:50am - 10:40am	**IL-11: Peter Lax Award Lecture** Location: **Neues Schloss – Weißer Saal**
	Recent progress on interaction energy minimizers **Ruiwen Shu**
10:40am - 11:10am	**Coffee Break**
11:10am - 12:00pm	**PL-06: Plenary Lecture** Location: **Neues Schloss – Weißer Saal**
	Multidimensional Riemann problems and hyperbolic conservation laws **Gui-Qiang George Chen**
12:00pm - 12:15pm	**Closing** Location: **Neues Schloss – Weißer Saal**

HYP2026 Book of Abstracts

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1 Prize Lectures

James Glimm Lecture

From real to ideal compressible fluids

Feireisl, Eduard

Institute of Mathematics - Czech Academy of Sciences Praha, Czech Republic

Friday, May 29, 09:00-09:50, Neues Schloss – Weißer Saal

The Euler system of gas dynamics is one of the iconic examples of systems of hyperbolic conservation laws. Recently, several disturbing fact concerning well posedness of this problem have been revealed. In the first part of the talk, the celebrated Glimm's method of numerical analysis, combined with the ill–posedness results based on the adaptation of convex integration, will be used to show density of the so–called wild data. Next, we discuss the possibility of a selection of a suitable weak solution. Strangely enough, the criteria of Dafermos and DiPerna based on maximality of the entropy production rate and the conventional vanishing viscosity limit give rise to rather different results. Finally, we propose a way how to identify the selected solutions by means of a numerical approximation.

Peter Lax Award Lecture

Recent progress on interaction energy minimizers

Shu, Ruiwen

University of Georgia, US

Friday, May 29, 09:50-10:40, Neues Schloss – Weißer Saal

Interaction energy minimizers appear natually in the study of collective dynamics in biological and social sciences. Despite the simple formulation, these minimizers may exhibit rich and unexpected pattern formation. The study of existence, uniqueness and qualitative behavior of minimizers is often challenging and requires the development of new theoretical tools. In this talk, I will discuss my recent results on the uniqueness of minimizers based on convexity theory, as well as the theoretical justification of annulus-shaped minimizers.

2 Plenary Lectures

Implosions in compressible Euler

Vicol, Vlad Cristian

Courant Institute School, New York University, USA

Monday, May 25, 10:00-10:50, M17.01

The compressible Euler equations are the fundamental model of gas dynamics. These PDEs admit a remarkable class of singularities known as implosions: a subset of the primary thermodynamic flow variables (density, pressure) become unbounded at a single point in space-time. Implosions are a much more dramatic kind of singularity than the more familiar shock waves. Understanding the existence and stability of such singularities is a fundamental problem. In this talk, we survey a number of recent results on Euler implosions.

Strong stability preserving time-evolution of hyperbolic PDEs: bounds and barriers

Gottlieb, Sigal

University of Massachusetts Dartmouth, USA

Tuesday, May 26, 09:00-09:50, M17.01

Strong stability preserving (SSP) time discretizations play a central role in the reliable numerical solution of hyperbolic partial differential equations, particularly in the presence of discontinuities, shocks, and sharp gradients. These methods are designed to inherit nonlinear stability properties—such as total variation diminishing (TVD), maximum principle preservation, or contractivity—from forward Euler under suitable time-step restrictions. However, the design of high-order SSP methods is fundamentally constrained by theoretical bounds and barriers that limit achievable order and allowable time steps. This talk will present an overview of the theory of SSP time integration, emphasizing both classical results and recent advances. We will discuss optimal SSP coefficients, order barriers for explicit and implicit Runge–Kutta and multistep methods, and extensions to multiderivative, additive, and integrating factor frameworks. We will also highlight emerging directions that aim to overcome traditional SSP barriers, including the use of multiderivative formulations.

High-order well-balanced schemes for balance laws: recent advances, challenges, and numerical results

Castro, Manuel J.

Universidad de Málaga, Spain

Wednesday, May 27, 09:00-09:50, M17.01

The development of high-order, well-balanced numerical methods for systems of balance laws remains an active area of research. Building upon the pioneering contributions of Bermúdez and Vázquez-Cendón (1994), A.Y. LeRoux (1994–1997), L. Gosse (2001–2002), and F. Bouchut (2004), numerous strategies have been developed over the past three decades. This presentation offers an overview of recent approaches to constructing high-order, well-balanced finite volume schemes, focusing specifically on the general mathematical framework introduced by M. Castro and C. Parés [2]. The cornerstone of these methods is a well-balanced reconstruction operator designed to preserve stationary solutions exactly.

The presentation will address two primary topics. First, we discuss the spatial discretization of complex 1D systems featuring singular source terms, particularly those involving the derivatives of discontinuous functions. In these cases, we apply the Generalized Hydrostatic Reconstruction technique [1], which effectively manages the numerical approximation of non-smooth stationary solutions. Second, we detail the development of high-order, well-balanced implicit and implicit-explicit (IMEX) schemes for hyperbolic balance laws [3]. These strategies are crucial for the efficient integration of systems characterized by stiff source terms or multiple scales. They ensure that the delicate balance between numerical fluxes and source term discretizations is strictly maintained without compromising computational efficiency.

Acknowledgements. M.J. Castro gratefully acknowledges the invaluable contributions and fruitful collaborations of his co-authors: C. Parés, M.L. Muñoz, T. Morales, C. Escalante, I. Gómez, C. Caballero, E. Pimentel, M. Dumbser, E. Gaburro, E.F. Toro, G. Russo, C. Chalons, C. Berthon, M. Ricchiuto, J.A. García, V. González, and S. Boscarino, among many others.

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[1] M.J. Castro, A. Pardo, and C. Parés. Well-balanced numerical schemes based on a generalized hydrostatic reconstruction technique. *Mathematical Models and Methods in Applied Sciences*, 17 (12): 2055–2113, 2007.

[2] M. J. Castro and C. Parés. Well-balanced high-order finite volume methods for systems of balance laws. *Journal of Scientific Computing*, 82(2):48, 2020.

[3] I. Gómez-Bueno, S. Boscarino, M.J. Castro, C. Parés and G. Russo. Implicit and semi-implicit well-balanced finite-volume methods for systems of balance laws. *Applied Numerical Mathematics*, 184: 18–48, 2023.

[4] I. Gómez-Bueno, M.J. Castro and C. Parés. High-Order Well-Balanced Methods for the Euler Equations of Gas Dynamics with Gravitational Forces and the Ripa Model. *Journal of Scientific Computing*, 102(3):77, 2025.

Preserving asymptotic limits at the discrete level

Puppo, Gabriella

Università di Roma La Sapienza, Italy

Wednesday, May 27, 11:10-12:00, M17.01

Many systems of hyperbolic partial differential equations depend on an adimensional parameter which triggers different phenomena according to its size. This is the case for instance of the Knudsen number in kinetic models or the Mach number in gas dynamics. In this talk, I will focus on numerical problems arising when dealing with the transition from compressible to incompressible regimes as the Mach number becomes small. There is an extensive body of research that has been devoted to this topic in the last twenty years. As we rely on numerical computations to approximate solutions and study their performance, we wish to develop schemes that preserve the asymptotic behaviour of the solution also at the discrete level. Low Mach and All Mach schemes are numerical methods which are designed to preserve the incompressible limit also at the discrete level.

Hyperbolic phenomena are characterized by different velocities, which are encripted in the eigenvalues of the Jacobian matrix of the flow. For gas dynamics, these are the acoustic waves which travel with the sound speed, the convective waves which move with the gas speed, but also elastic waves, if one considers more complex materials. We can also have several acoustic speeds for mixtures composed of several fluids, with different equations of state. When we study these phenomena from a numerical point of view however, we must choose a grid, with a typical mesh width h, and a time step Δt. This induces a "numerical speed" $a = h/\Delta t$. Whether we use uniform or adaptive grids, at any point we typically have only one "numerical speed" to represent phenomena that instead are far richer. An important aspect of a numerical scheme than becomes ensuring the possibility of choosing which phenomena the scheme should best capture, knowing that we have a single numerical speed at our disposal. Low Mach schemes answer this need: they are of interest when convective waves are the waves of interest, and acoustic waves can be smeared out.

In this talk, I will present ideas we developed for low Mach schemes applied to gas dynamics, elastic materials and two phase flow. I will also hint at a fully implicit scheme designed to target one specific speed, which can be chosen by the user.

Acknowledgements. Cofunded by MUR, project PRIN2022JH87B4.

Structure and asymptotic preserving schemes for the quasineutral limit of the Vlasov-Poisson system

Filbet, Francis

Université de Toulouse, Institut de Mathématiques de Toulouse, France

Thursday, May 28, 09:00-09:50, M17.01

In this presentation, we introduce novel structure-preserving methods tailored for the nonlinear Vlasov-Poisson-Fokker-Planck (VPFP) system. By reformulating the VPFP model as a hyperbolic system via a Hermite expansion in velocity space, we develop discretization schemes that incorporate both discontinuous Galerkin and mixed finite element techniques for the coupled equations. We establish exponential relaxation to equilibrium through discrete hypocoercivity arguments, ensuring uniform long-time stability with respect to discretization parameters. Our schemes preserve key physical invariants and maintain the L^2 variational structure of the linearized model. Furthermore, we analyze the asymptotic behavior in the quasineutral regime, proving convergence of the electric field to its quasineutral limit with optimal error estimates. Nu- merical simulations validate the accuracy, stability, and asymptotic preservation of our approach, demonstrating its effectiveness in capturing the long-term dynamics of quasineutral plasmas. This work provides a robust framework for the numerical analysis of kinetic models in plasma physics, with rigorous guarantees in both the long-time and quasineutral regimes.

Multidimensional Riemann problems and hyperbolic conservation laws

Chen, Gui-Qiang George

University of Oxford, United Kingdom

Friday, May 29, 11:10-12:00, Neues Schloss – Weißer Saal

In this talk, we first discuss the underlying connections between multidimensional Riemann problems—focusing on their global patterns and structures—and general entropy solutions of nonlinear hyperbolic systems of conservation laws. We then present recent progress in the rigorous analysis of several longstanding two-dimensional Riemann problems, both initial and lateral, involving transonic shock waves for the Euler equations of potential flow. In particular, we focus on the four-shock Riemann problem for the Euler equations for potential flow, as a representative example, to illustrate how these problems can be reformulated and solved as free boundary problems, with transonic shock waves serving as free boundaries for the corresponding nonlinear conservation laws of mixed elliptic–hyperbolic type. We further address several physically significant lateral Riemann problems, including Prandtl's reflection problem, von Neumann's shock reflection-diffraction problem, and Lighthill's diffraction problem. Moreover, we present different regularity properties of Riemann solutions to the compressible Euler equations for both potential and isentropic flows. We also discuss further multidimensional Riemann problems and related shock wave problems for nonlinear hyperbolic systems of conservation laws, if time permits.

3 Invited Lectures

Structure-preserving neural surrogates for uncertainty quantification in kinetic equations

Pareschi, Lorenzo

Heriot Watt University, United Kingdom

Monday, May 25, 11:20-12:00, M17.01

Kinetic equations such as the Boltzmann and Landau equations describe nonequilibrium regimes in rarefied gases, plasmas, and related multiscale systems. Their numerical approximation has led to a mature high-order methodology, combining spectral methods for collisional operators, high-order finite-volume or discontinuous Galerkin discretizations for transport, and implicit-explicit time integration in stiff regimes, with conservation and asymptotic consistency built into the numerical design. Uncertainty quantification changes the computational perspective: the challenge is no longer a single accurate kinetic simulation, but the repeated evaluation of high-fidelity models over random inputs and across regimes. For kinetic equations, each such evaluation already involves a high-dimensional and stiff phase-space computation, so that direct sampling rapidly becomes a major computational bottleneck. This makes machine learning relevant as part of a multifidelity numerical strategy. Neural surrogates are used as control variates, where their role is governed by approximation error, evaluation cost, and correlation with the high-fidelity solver. Within this framework, their impact on the final statistical estimator can be quantified by rigorous error estimates. I will discuss asymptotic-preserving and structure-preserving neural architectures, with emphasis on positivity, conservation of moments, entropy consistency, and the correct fluid limit. Micro-macro decompositions and tensorized representations of velocity moments are used to obtain genuine computational gains. Examples for the Boltzmann equation and the Vlasov-Poisson-Landau system illustrate how high-order kinetic solvers and neural surrogates can be combined for reliable uncertainty quantification across kinetic and fluid regimes. This is joint work with Wei Chen (Xiamen University) and Giacomo Dimarco (University of Ferrara).

A Lax-type framework for fluid models: convergence and error estimates

She, Bangwei

Capital Normal University, China, People's Republic of

Tuesday, May 26, 09:55-10:35, M17.01

In this talk, we present recent advances of Lax-type convergence theorems for numerical approximations of fluid flows. The guiding principle is a nonlinear counterpart of the classical implication "stability + consistency $\Longrightarrow$ convergence", where stability includes discrete energy/entropy control and positivity preservation of thermodynamic variables—properties typically embedded in structure-preserving numerical schemes. Starting from the a priori bounds encoded in stability, we derive a suitable weak consistency formulation. Then passing to the limit yields convergence of numerical solutions to dissipative solutions. Moreover, applying the weak–strong uniqueness principle leads to convergence towards the classical solution on its lifespan, and provides quantitative convergence rates towards a smooth solution. We will discuss applications of this framework to the compressible Navier–Stokes equations, the incompressible Euler equations, and related fluid models. In particular, we address three prototypical scenarios:

(i) convergence to dissipative solutions when classical solutions are not available;

(ii) convergence to the classical solution on the time interval where it exists;

(iii) convergence rates with respect to a smooth solution.

This is joint work with E. Feireisl, M. Lukáčová-Medviďová, and H. Mizerová.

Learn to evolve: self-supervised neural operator for nonlinear gradient flows

Wang, Li

University of Minnesota, United States of America

Tuesday, May 26, 09:55-10:35, M17.02

The Jordan-Kinderlehrer-Otto (JKO) scheme provides a stable variational framework for computing Wasserstein gradient flows, but its practical use is often limited by the high computational cost of repeatedly solving the JKO subproblems. We propose a self-supervised approach for learning a JKO solution operator without requiring numerical solutions of any JKO trajectories. The learned operator maps an input density directly to the minimizer of the corresponding JKO subproblem, and can be iteratively applied to efficiently generate the gradient-flow evolution. A key challenge is that only a number of initial densities are typically available for training. To address this, we introduce a Learn-to-Evolve algorithm that jointly learns the JKO operator and its induced trajectories by alternating between trajectory generation and operator updates. As training progresses, the generated data increasingly approximates true JKO trajectories. Meanwhile, this Learn-to-Evolve strategy serves as a natural form of data augmentation, significantly enhancing the generalization ability of the learned operator. Numerical experiments demonstrate the accuracy, stability, and robustness of the proposed method across various choices of energies and initial conditions.

Geometric Quasi-Linearization (GQL) and structure-preserving analysis: from a general framework to unveiling algebraic–differential relations in MHD

Wu, Kailiang

Department of Mathematics and Shenzhen International Center for Mathematics, Southern University of Science and Technology (SUSTech), Shenzhen, Guangdong 518055, P.R. China

Tuesday 26, 11:20-12:00, M17.01

Preserving invariant regions (e.g., positivity of density and pressure, and the subluminal-velocity constraint in relativity) is fundamental to the physical consistency and mathematical well-posedness of hyperbolic conservation laws. However, developing high-order schemes that strictly respect nonlinear constraints remains a significant challenge for hyperbolic systems and computational fluid dynamics.

In the first part of this talk, we introduce the Geometric Quasi-Linearization (GQL) framework. Building on key insights from convex geometry, GQL transforms arbitrary nonlinear convex constraints into *equivalent* linear constraints by introducing free auxiliary variables. Crucially, GQL reveals a geometric insight: nonlinear constraints in the physical space are, in essence, linear when lifted to a higher-dimensional space. We establish the theoretical foundation of this framework and propose three effective construction methods, providing a unified approach to nonlinear invariant-domain-preserving analysis and design.

In the second part, we apply GQL to the structure-preserving analysis of (ideal/relativistic) compressible magnetohydrodynamics (MHD), unveiling an intrinsic algebraic–differential coupling between thermodynamic constraints (pressure positivity) and the involution constraint (divergence-free magnetic field) at both the numerical and PDE levels. We prove that the preservation of algebraic bounds is generally conditioned on the discrete divergence-free structure. By deriving the exact compatibility condition, we construct high-order schemes that achieve synergistic preservation of both algebraic and differential constraints. The robustness of these methods is demonstrated through extreme simulations, including low plasma beta ($\approx 10^{-10}$) blast waves and astrophysical jets with Mach numbers up to 10^6.

This talk is based on joint work with Prof. Chi-Wang Shu (Brown University), Prof. Huazhong Tang (Peking University), Prof. Rémi Abgrall (University of Zurich), and (current and former) students and postdocs of my research group, including Dr. Huihui Cao, Dr. Shengrong Ding, Dr. Haili Jiang, Dr. Mengqing Liu, Mr. Dongwen Pang, Ms. Manting Peng, Mr. Linfeng Xu, and Dr. Caiyou Yuan.

Two-dimensional Euler flows beyond Yudovich

Brué, Elia
Bocconi University, Italy

Tuesday, May 26, 11:20-12:00, M17.02

I will survey some recent advances in the study of weak solutions to the two-dimensional incompressible Euler equations. The focus will be on solutions with L^p-integrable vorticity, a natural class connecting the classical Yudovich theory with more singular regimes. After reviewing some background and mathematical context, I will discuss recent developments revealing new flexibility phenomena at low integrability.

Numerical discretization of conservation laws using Young measures

Herty, Michael
RWTH Aachen, Germany

Wednesday, May 27, 09:55-10:35, M17.01

A novel structure-preserving numerical method to solve random hyperbolic systems of conservation laws is presented. The method uses a concept of generalized, measure-valued solutions to random conservation laws. This yields a linear partial differential equation with respect to the Young measure and allows to compute the approximation based on linear programming problems. We analyse structure-preserving properties of the derived numerical method and discuss its advantages and disadvantages. Numerical results for scalar, discontinuous conservation laws as well as for multi-d problems are presented.

Singular limits in some nonlinear continuity equations

Crippa, Gianluca
University of Basel, Switzerland

Wednesday, May 27, 09:55-10:35, M17.02

In this talk, I will review recent results obtained in several joint works on singular limits for nonlinear partial differential equations with a continuity equation structure. I will present results on nonlocal approximations of continuity equations, where the advecting velocity field depends on the solution through a convolution with a family of kernels converging to a Dirac delta, as well as on vanishing viscosity approximations and related regularizations, including schemes with a numerical flavor. In particular, I will discuss consequences for the vanishing viscosity problem for the two-dimensional Euler equations. I will also show how these approximations converge to solutions with stronger properties than general weak solutions. Finally, I will briefly address quantitative aspects of these singular limits, including convergence rates.

Recent advances in subcell based limiting for discontinuous Galerkin methods with guarantees

Gassner, Gregor

University of Cologne, Germany

Thursday, May 28, 09:55-10:35, M17.01

In this talk, we present recent advances in subcell-based limiting for high-order spectral element Discontinuous Galerkin (DG) methods. The central idea is to construct an equivalent finite-volume formulation of the high-order DG scheme on a subcell Legendre–Gauss–Lobatto grid. On this subcell grid, we design a provably invariant-domain-preserving (IDP) low-order finite-volume method that acts as a robust companion scheme.

Both schemes are compatible in the sense that convex combinations of the original high-order scheme and the robust low-order scheme can be defined locally, on a pointwise basis. By employing flux limiters from the flux-corrected transport (FCT) literature, we can enforce mathematical guarantees on the resulting solution under a CFL-type condition, such as entropy inequalities and physically relevant bounds (e.g., positivity of density and pressure).

Recent developments extend this framework to adaptive mesh refinement (AMR). A key ingredient is a provably IDP low-order scheme on the subcells that can handle hanging nodes in nonconforming meshes. A second important ingredient is a controlled way to transfer polynomial data to coarse grids, where we use a polynomial type filter/limiter based on work of Zhang and Shu. Another recent advancement is the extension of the general approach to hyperbolic systems with non-conservative terms.

All new schemes and advances are implemented in the open source code Trixi.jl that is based on the Julia programming language. The developments are carried out in close collaboration with Benjamin Bolm, Andrés Rueda-Ramîrez, Andrew Winters, Patrick Ersing, and Dmitri Kuzmin.

Numerical solutions of hyperbolic equations – wave breaking leads the way

Grunert, Katrin

NTNU - Norwegian University of Science and Technology, Norway

Thursday, May 28, 09:55-10:35, M17.02

Solutions of hyperbolic equations might enjoy wave breaking in finite time. This means that classical solutions in general do not exist globally, but only locally in time, since a directional derivative might become unbounded pointwise, while the solution itself remains bounded and continuous. In addition, energy may concentrate on sets of measure zero when wave breaking occurs. As a consequence the prolongation of solutions beyond wave breaking is non-unique and depends heavily on how the concentrated energy is manipulated.

In this talk, we will first focus on the Hunter–Saxton equation and on α-dissipative solutions, i.e., solutions where the energy is manipulated at breaking time by taking out an α-part of the concentrated energy. Guided by analytical methods and results, we will present a numerical method for α-dissipative solutions, which can handle any $\alpha \in [0, 1]$ and in particular the two most prominent types of solutions: conservative, i.e., $\alpha = 0$, and dissipative, i.e., $\alpha = 1$. Convergence is obtained for any admissible initial data, while a convergence rate is derived under some additional constraints on the initial data.

Second, we will turn our attention towards the nonlinear variational wave equation, which has the Hunter–Saxton equation as an asymptotic approximation. We will highlight which additional difficulties arise, due to considering a second oder hyperbolic equation, and how these influence the underlying methods and techniques.

4 Contributed Lectures

P1-1 Well-posedness and Regularity

Variational formulation of entropic solutions to nonlinear conservation laws

Tadmor, Eitan
University of Maryland, USA

Monday, May 25, 14:00-14:25, M17.12 – 1.005

Entropy functions played a key role in the development of mathematical theory for hyperbolic conservation laws. Yet it is known that the entropy inequality cannot serve as a selection principle in nonlinear systems of conservation laws in more than one spatial dimension. The notion of entropy, which is intimately connected with symmetry, is an extension imposed on nonlinear systems conservation laws. In this context, K. O. Friedrichs asked whether the assumed symmetries can also be derived. In this lecture I will present a variational formulation which addresses the question raised by Friedrichs and offers a possible resolution to the question of uniqueness.

Temperature-driven turbulence in compressible fluid flows

Yuan, Yuhuan
Nanjing University of Aeronautics and Astronautics, China

Monday, May 25, 14:25-14:50, M17.12 – 1.005

We study the long-time behaviour of the temperature-driven compressible flows. We show that numerical solutions of a structure-preserving finite volume method generate a discrete attractor that consists of entire discrete trajectories. Further, we prove the convergence of discrete attractors to their continuous counterparts. Theoretical results are illustrated by extensive numerical simulations of the well-known Rayleigh–Bénard problem. The numerical results also indicate the validity of the ergodic hypothesis and imply that a non-zero Reynolds stress persist for long time. Finally, we also observe that any invariant measure is of Gaussian type in sharp contrast with the conjecture proposed by [Glimm et al., SN Applied Sciences, 2:paper no.2160, 2020].

[1] E. Feireisl, M. Lukáčová-Medviďová, B. She and Y. Yuan. Temperature-driven turbulence in compressible fluid flows. Preprint 2026.

[2] E. Feireisl, M. Lukáčová-Medviďová, B. She and Y. Yuan. Convergence of a finite volume method to weak solutions to the compressible Navier–Stokes–Fourier system. Preprint 2026.

[3] E. Feireisl, M. Lukáčová-Medviďová, B. She and Y. Yuan. Convergence of numerical methods for the Navier-Stokes-Fourier system driven by uncertain initial/boundary data. *Found. Comput. Math.*, **25**, 1507-1559, 2025.

The energy-variational framework for binormal curvature flow

Eiter, Thomas; Lasarzik, Robert; Śliwiński, Marcel

Freie Universität Berlin, Germany

Monday, May 25, 14:50-15:15, M17.12 – 1.005

Strongly localized vortical structures are frequently observed in fluid dynamics. A classical model for the motion of vortex filaments is the equation for binormal curvature flow. However, this differential equation for the parametrization of a closed, oriented curve fails to capture the formation of corners and kinks, as well as the finite-time blow-up of smooth solutions. This limitation motivates a relaxation of the solution concept. We begin by considering a weak formulation, which naturally extends to divergence-free measures instead of curves. In this framework, the length of the curve, which is an invariant preserved by smooth solutions and serving as an energy, is replaced with the total variation of the measure. Based on this, we introduce he notion of energy-variational solutions, a concept that augments the combination of the energy inequality and the weak formulation with an additional defect term. A general result for abstract evolution equations [1] enables to derive the global-in-time existence of energy-variational solutions in this framework. Additionally, we discuss a weak-strong uniqueness principle that guarantees that energy-variational solutions coincide with classical solutions as long as the latter exist.

Acknowledgements. This project was funded by DFG through grant CRC 1114 "Scaling Cascades in Complex Systems", Proj. #235221301, Project YIP, and grant SPP 2410 "Hyperbolic Balance Laws in Fluid Mechanics: Complexity, Scales, Randomness (CoScaRa)", Proj. #525941602.

[1] T. Eiter, R. Lasarzik, and M. Śliwiński. Existence and selection of solutions in the energy-variational framework with applications in fluid dynamics, arXiv:2601.20455, 2026.

A Liouville-type theorem for 2×2 systems of hyperbolic conservation laws and applications

Talamini, Luca; Ancona, Fabio; Marconi, Elio

SISSA (Trieste), Italy

Monday, May 25, 15:15-15:40, M17.12 – 1.005

The pioneering work of Tartar and DiPerna established the existence of entropy solutions to systems of two conservation laws in the $\mathbf{L}^\infty$ setting by adapting the compensated compactness method. However, no regularity properties are known for these solutions. As a first step in this direction, we discuss a Liouville-type theorem for global isentropic solutions of 2x2 genuinely nonlinear systems, showing that every such solution must be a constant. Some applications of the Liouville type theorem include VMO (vanishing mean oscillation) regularity outside a set $J \subset \mathbb{R}^+ \times \mathbb{R}$ with Hausdorff dimension less then 1 (the candidate shock set), and a strong time regularity result, valid for general entropy solutions in $\mathbf{L}^\infty$, which shows that bounded entropy solutions lie in $C^0(\mathbb{R}^+, \mathbf{L}^1_{loc}(\mathbb{R}))$. The arguments are based on a kinetic formulation and on a Lagrangian representation at the kinetic level.

P1-2 Well-Posedness and Regularity

Global well-posedness and asymptotic behavior of large strong solutions to viscous compressible fluids with variable coefficients

Li, Yachun

Shanghai Jiao Tong University, PR China

Monday, May 25, 16:00-16:25, M17.12 – 1.005

In this talk, we discuss the 3D Navier-Stokes equations with density-dependent or temperature-dependent coefficients in the whole space. We establish the global existence of large strong solution when the initial density and the initial temperature are linearly equivalent to some large constant states. Moreover, the optimal decay rates of the solution to its associated equilibrium are established. This is a joint work with Peng Lu and Zhaoyang Shang.

Global well-posedness for the degenerate compressible Navier-Stokes equations with large data of spherical symmetry

Zhang, Jiawen; Chen, G. Gui-Qiang; Zhu, Shengguo

Shanghai Jiao Tong University, P. R. China

Monday, May 25, 16:25-16:50, M17.12 – 1.005

A longstanding open problem in the theory of compressible Navier-Stokes equations is whether regular spherically symmetric flows can develop singularities, such as cavitation or implosion, in finite time. In this talk, I will present our recent works [1, 2] that provide a definitive answer to this question for the barotropic compressible Navier-Stokes equations with degenerate, density-dependent viscosity, as in the viscous shallow water system (*i.e.*, Saint-Venant system).

We consider two of its most challenging settings — the Cauchy problem (with far-field vacuum or strictly positive initial density) and the vacuum free boundary problem (in which vacuum appears in an open set) — in two and three space dimensions. In both cases, we prove that solutions remain globally regular and cannot undergo cavitation (inside the fluid) or implosion, thereby establishing the global well-posedness of classical solutions with large spherically symmetric initial data. The methodology developed here should also be useful for solving other related nonlinear partial differential equations involving similar difficulties.

[1] G.-Q. Chen, J. Zhang, and S. Zhu. Global regular solutions of the multidimensional degenerate compressible Navier-Stokes equations with large initial data of spherical symmetry, arXiv:2512.18545, 2025.

[2] G.-Q. Chen, J. Zhang, and S. Zhu. Global well-posedness of the vacuum free boundary problem for the degenerate compressible Navier-Stokes equations with large data of spherical symmetry, arXiv:2601.06620, 2026.

Global existence and nonlinear stability of finite-energy solutions of the compressible Euler-Riesz equations with large initial data of spherical symmetry

Difan Yuan

Beijing Normal University, China

Monday, May 25, 16:50-17:15, M17.12 – 1.005

We establish the global existence and unconditional nonlinear stability of finite-energy solutions for the multidimensional compressible Euler-Riesz equations with large, spherically symmetric initial data. This is achieved via an inviscid limit from the Navier-Stokes-Riesz equations. A key finding is the absence of concentration at the origin, even for singular attractive potentials. The analysis, which covers Riesz and logarithmic potentials, requires novel estimates to handle nonlocal singularities beyond the Coulomb case. Steady states and their stability are characterized using variational and concentration compactness methods. This is a joint work with J.A. Carrillo, S.R. Charles and G.Q. Chen.

On weak solutions to self-organized systems of Euler-type with non-constant interaction kernel

Cipollone, Gianmarco; Amadori, Debora; Christoforou, Cleopatra;

University of L'Aquila, Italy

Monday, May 25, 17:15-17:40, M17.12 – 1.005

In this talk, we consider an Euler-type model for flocking, motivated by the description of collective behavior in systems of interacting agents. The model describes the evolution of macroscopic density and velocity, and features a pressure term together with an integrable, positive interaction kernel.

We focus on the evolution of entropy weak solutions that are periodic in space, and on their long-time behavior. Assuming the existence of such solutions, we show that their L^2 norm decays in time towards the average values. The approach makes use of entropy methods and it is inspired by the work of C.M. Dafermos [2] on the long-time behavior for hyperbolic systems with dissipative source.

The global existence of entropy weak solutions has been established in [2], for the case of linear pressure and constant kernel. We address the issue of global existence result in the more general setting of a non-constant, integrable kernel by means of a wave-front tracking approach.

[1] D. Amadori, F. A. Chiarello and C. Christoforou. Euler-flocking system with nonlocal dissipation in 1D: periodic entropy solutions. *Journal of Hyperbolic Differential Equations*, 2026.

[2] C. M. Dafermos. Periodic BV solutions of hyperbolic balance laws with dissipative source. *Journal of Mathematical Analysis and Applications*, 428(1):405–413, 2015.

Well posedeness for a tropical climate phase transition model

Cianfarani Carnevale, Giada; Donatelli, Donatella; Spirito, Stefano

University of L'Aquila, Italy

Monday, May 25, 17:40-18:05, M17.12 – 1.005

In this work, we study a phase transition model in tropical atmospheric dynamics, based on the primitive equations for velocity, temperature, and specific humidity. The main analytical difficulty comes from the presence of a discontinuous multivalued nonlinear term in the temperature and humidity equations, modelling the onset of precipitation under supersaturation conditions, as in the framework introduced in [1].

To overcome this issue, we introduce a suitable regularized formulation, which ensures the existence and uniqueness of approximate solutions. By combining uniform a priori estimates with classical compactness arguments, we then prove the existence and uniqueness of a strong solution to the original model on the whole space $\mathbb{R}^2$, without adding any artificial viscosity term in the humidity equation.

Our result provides a rigorous justification of the model in an unbounded domain, extending the analysis of [1], which is restricted to bounded domains, and incorporating the more realistic precipitation nonlinearity of the latter into the simpler analytical framework proposed in [2].

1 Coti Zelati, M., et al. The primitive equations of the atmosphere in presence of vapour saturation. Nonlinearity 28(3), 2015.

2 Li, J.; Titi, E. S. A tropical atmosphere model with moisture: global well-posedness and relaxation limit. Nonlinearity 29 (2016), no. 9, 2674–2714.

P1-3 Well-Posedness and Regularity

Regularity of velocity averages in kinetic equations with heterogeneity

Karlsen, Kenneth

University of Oslo, Norway

Tuesday, May 26, 14:00-14:25, M17.12 – 1.005

In joint work with Marko Erceg (Zagreb) and Darko Mitrović (Montenegro/Vienna), we establish new regularity results for kinetic equations with spatially heterogeneous drift. Under a quantitative non-degeneracy condition, we show that velocity averages of weak solutions possess fractional Sobolev regularity in space and time, under minimal integrability assumptions. Our results apply to solutions that are integrable to any power strictly larger than one and allow for rough source terms. Applications include conservation laws with discontinuous fluxes and regularity of the density in isentropic gas dynamics.

[1] M. Erceg, K. H. Karlsen and D. Mitrović.Regularity of velocity averages in kinetic equations with heterogeneity, submitted, 2025.

On the convergence of quasilinear viscous approximations using compensated compactness and kinetic formulation

Mondal, Ramesh

University of Kalyani, India.

Tuesday, May 26, 14:25-14:50, M17.12 – 1.005

We use the method of Compensated Compactness and Kinetic Formulation to show that the almost everywhere limit of quasilinear viscous approximations is the unique entropy solution (in the sense of *F. Otto* of the corresponding scalar conservation laws on a bounded domain in $\mathbb{R}^d$, where the viscous term is of the form $\varepsilon div\,(B(u^\varepsilon)\nabla u^\varepsilon)$.

Effective equations for a compressible liquid-vapor flow model

Wendt, Florian; Rohde, Christian

University of Stuttgart, Germany

Tuesday, May 26, 14:50-15:15, M17.12 – 1.005

In this talk, we derive and justify rigorously a new effective model for a compressible viscous liquid-vapor flow on a spray-like scale. The justification relies on describing the liquid-vapor flow by a parabolic relaxation formulation of the compressible Navier–Stokes–Korteweg system with a non-monotone pressure function. In the first part of this talk, we introduce the class of finite energy weak solutions to the initial-boundary value problem corresponding to the relaxation model. In the second part of this talk, we consider a sequence of initial data, where the sequence of initial densities is assumed to be highly oscillating mimicking the large number of phase boundaries initially. For a corresponding sequence of finite energy weak solutions, we anticipate that the effective equations are found in the limit of infinitely many initial phase changes. By interpreting the densities as Young measures, we prove the convergence of the sequence of solutions to the effective model. The effective model consists of a probabilistic part for the limit Young measure

and a deterministic part for the fluid's hydrodynamic quantities. In the last part of this talk, we rewrite the equation for the Young measure into a kinetic equation for its cumulative distribution function so that the resulting equations are accessible by standard approximation methods.

Hypersonic flow and Radon measure-valued solution

Qu, Aifang

Shanghai Normal University, China

Tuesday, May 26, 15:15-15:40, M17.12 – 1.005

Hypersonic flows usually refer to those with Mach number larger than 5 and feature ultra-thin shock layers and Mach number independence principle. Concentrated singularities arise as Mach number approaches infinity and the adiabatic exponent γ goes to 1. The Newtonian sine-squared law has been obtained by physics from this limit and it is valid and widely used in hypersonic aerodynamics. We give a mathematical insight of this kind of phenomena and develop the concept of weak solution of conservation laws proposed by P. Lax to Radon measure-valued solution for compressible Euler equations.

P1-4 Well-Posedness and Regularity

Nonlinear sound waves in compressible Euler

Young, Robin

University of Massachusetts, USA

Tuesday, May 26, 16:00-16:25, M17.12 – 1.005

We prove the generic existence of "pure tone" nonlinear sound waves of all frequencies on bounded domains. These are smooth, time periodic, oscillatory solutions of the full compressible Euler equations subject to natural "acoustic" boundary conditions. Being perturbations of solutions of the linear wave equation, they provide a rigorous justification for the theory of Acoustics. This is an extension of the one-dimensional theory developed with Blake Temple.

On Lions density patch at the critical level

Violini, Alessandro; Škondrić, Stefan

Univeristy of Basel, Switzerland

Tuesday, May 26, 16:25-16:50, M17.12 – 1.005

We study the two–dimensional incompressible Navier–Stokes equations for a two–fluid configuration. More precisely, we consider initial densities of the form

$$\rho_0 = \mathbf{1}_D,$$

where $D \subset \mathbb{R}^2$ is a bounded domain with Lipschitz boundary. This corresponds to a homogeneous incompressible fluid occupying the region D and surrounded by vacuum. This configuration corresponds to the classical density patch problem raised by Lions in his monograph *Mathematical Topics in Fluid Mechanics*, where the main question is whether the geometric regularity of the patch is preserved by the evolution.

We prove that if the initial velocity satisfies $u_0 \in \dot{B}^0_{2,1}(\mathbb{R}^2)$, which is a critical space, and is divergence free, then the system admits a unique global solution such that

$$\nabla u \in L^1\big((0,\infty); L^\infty(\mathbb{R}^2)\big).$$

This property ensures the well-definedness and stability of the flow map transporting the density. As a consequence, the patch structure is preserved for all times:

$$\rho(t) = \mathbf{1}_{D_t},$$

where D_t is a domain with Lipschitz boundary for every $t > 0$.

As a consequence of the global bounds and of the dissipation mechanism induced by viscosity, we also describe the large-time behaviour of the patch. We show that the transported domains asymptotically converge, up to translation, to a limiting Lipschitz domain. More precisely, if $X(t,\cdot)$ denotes the flow map associated with the velocity field and M the average initial momentum, then there exists a bi-Lipschitz and measure-preserving map X_∞ such that

$$X(t,\cdot) - Mt \to X_\infty(\cdot) \quad \text{in } L^\infty(D).$$

In particular, the domains $D_t - Mt$ converge to a limiting domain D_∞. This shows that viscosity progressively damps the internal deformations of the patch, while the conservation of momentum leaves only a rigid translation in the asymptotic regime.

Our result provides a positive answer to the density patch problem in the critical regularity framework.

Acknowledgements. Stefan Škondrić and Alessandro Violini are supported by the Deutsche Forschungsgemeinschaft (DFG) in the Priority Programme SPP 2410 "Hyperbolic Balance Laws in Fluid Mechanics: Complexity, Scales, Randomness" (CoScaRa).

Global small solutions to the 2D non-resistive compressible MHD system near an equilibrium

Zhu, Yi

East China University of Science and Technology, China, People's Republic of

Tuesday, May 26, 16:50-17:15, M17.12 – 1.005

This talk focuses on the compressible magnetohydrodynamic (MHD) equations without magnetic diffusion in $\mathbb{R}^2$. We present a systematic approach to establish the global existence of smooth solutions when the initial data is close to a background magnetic field. To overcome key challenges arising from anisotropy and the critical nature of the system, we fully exploit the underlying dissipative structure and introduce several delicately constructed dissipative quantities. Compared with the previous works, we solve this problem by pure energy estimate without extra help from Sobolev spaces of negative index.

Global existence/non-existence of classical solution for isentropic Euler equations with time dependent damping

Chhatria, Balakrishna

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Tuesday, May 26, 17:15-17:40, M17.12 – 1.005

In this work, we are concerned with the existence and non-existence of classical solution for the Cauchy solution of the isentropic Euler equation with time dependent damping and a general pressure term. For the existence of solution to the Cauchy problem we obtain uniform bounds on state variables and bounds on the derivative of those under suitable assumptions on initial data. In addition, we have provided scenarios where the classical solution will blow-up.

Well-posedness and numerical approximation of nonlinear conservation laws with hysteresis

Moreti, Stefan; Bagagiolo, Fabio; Goatin, Paola

University of Trento, Italy

Tuesday, May 26, 17:40-18:05, M17.12 – 1.005

We present a study of the Cauchy problem for a class of scalar conservation laws where the classical structure of the equation is coupled with a rate-independent hysteresis effect

$$\partial_t u + \partial_t \mathcal{F}(u) + \partial_x f(u) = 0.$$

Here, $\mathcal{F}$ represents a hysteresis operator, namely the Play or the Preisach operator, which introduces a non-local dependence on the history of the input u. Such models can capture phenomena

in several applications and pose challenges to the classical theory of hyperbolic equations, due to the non-locality of the memory term and its lack of stability under $\mathbf{L}^1$ convergence of inputs.

The analysis begins in [1] with the Play operator as a prototype hysteresis model and with the simpler case of linear flux $f(u) = u$. First, an appropriate notion of entropy weak solution, especially for what concerns the hysteresis relationship, is established. Then, the Riemann problem and the interactions between shock waves are studied in detail, and the existence of weak entropy solutions is proved via the Wave-Front Tracking method. The entropy condition guarantees uniqueness within the class of entropy-admissible solutions.

This theory is then extended in [2] to the case of $\mathcal{F}$ equal to the Preisach hysteresis operator, which generalizes the Play model by representing the output as a weighted superposition of elementary hysterons. This study hence requires the treatment of a continuum of internal variables, which increases the complexity of the problem.

Finally, in [3] we also address the more general case with convex flux functions $f(u)$ coupled with the Play operator. For this problem, a different approach is adopted: after solving the Riemann problem, a Godunov-type finite volume numerical scheme is developed. The convergence of the scheme for BV initial data provides a proof of existence, while stability estimates follow from the entropy condition ensuring the uniqueness and well-posedness of the problem. This theory can be also adapted to the case of linear flux $f(u) = u$, providing an alternative proof of existence.

These results provide a study of the equation with hysteresis within the framework of scalar conservation laws.

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P1-5 Well-Posedness and Regularity

Hölder continuity for multi-d balance laws

Caravenna, Laura; Cliffe, Alexander; Marconi, Elio; Pinamonti, Andrea

Università degli Studi di Padova, Italy

Thursday, May 28, 16:00-16:25, M17.12 – 1.005

What happens to solutions to balance laws that do not have discontinuities?

Surprisingly, continuous solutions to scalar balance laws can exhibit very irregular behavior from the Euclidean viewpoint—even failing to be Hölder continuous of any exponent when the flux lacks sufficient nonlinearity. However, when the flux is nonlinear, remarkably continuity improves to Hölder continuity. In this talk, I'll discuss how the degree of flux nonlinearity quantitatively determines the Hölder regularity of continuous isentropic solutions to multidimensional scalar balance laws with bounded source terms

$$u_t + \mathrm{div}_x f(u) = g, \qquad t \in \mathbb{R},\ x \in \mathbb{R}^d.$$

Exploiting the kinetic formulation, we establish that solutions are $\alpha/(2d)$-Hölder continuous under general α-nonlinearity assumptions, with an improved exponent of $1/(2d)$ for maximally nondegenerate fluxes such as the multidimensional Burgers equation $f(u) = (u^2, u^3, \ldots, u^{d+1})$. This extends previous one-dimensional results to the genuinely multidimensional setting and reveals the subtle relationship between nonlinear flux structure and such solution regularity.

Joint work with Fabio Ancona, Elio Marconi, and Alexander Cliffe

Regularity of Lagrangian flows associated to singular vector fields

Borrin, Henrique; Nariyoshi, João Fernando

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Thursday, May 28, 16:25-16:50, M17.12 – 1.005

We shall present the extension of the regularity for Lagrangian flows obtained in [1] previously only known for Sobolev vector fields $W^{1,p}$ for $p > 1$, or more generally for vector fields whose derivative have modulus of integrability $L \log L$. Our extension completes the range $W^{1,1}$ up to the latter integrability of the derivative, that is, we consider vector fields whose derivative has a convex, superlinear and increasing modulus of integrability, and we prove that up to a set of small measure, the associated Lagrangian flow is at least uniformly continuous in spatial variables. Moreover, the better the modulus of integrability of the vector field, then so is the modulus of continuity of of the associated flow, with $L \log L$ being the critical integrability for which the flows are (uniform in time) Lipschitz. Finally, we prove that if the modulus of integrability is smooth, the modulus of continuity of the associated flow is optimal.

Acknowledgements. Supported by FAPESP through grant 2024/21041-1.

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Oleinik inequality for balance laws

Harb, Rida; Junca, Stéphane

Université Côte d'Azur, Laboratoire Jean-Alexandre Dieudonné, France

Thursday, May 28, 16:50-17:15, M17.12 – 1.005

The classical one-sided Oleinik inequality is a cornerstone in the theory of scalar conservation laws with uniformly convex flux, i.e., $f'' \geq c_0 > 0$ for some constant c_0 [1]. In the spirit of this inequality, we establish a generalized Oleinik-type estimate for the unique entropy solution u of a scalar balance law with strictly convex flux f and nonlinear source term $g(u)$ [2]. Our proof is based on a *generalized* wave-front tracking algorithm, which extends the classical wave-front tracking schemes for scalar conservation laws [3, 4, 5] to the present balance law. The flux f is allowed to be degenerately convex, in the sense that its second derivative f'' may vanish on an infinite set while f remains strictly convex. Under a generic transversal condition linking the flux and the source term, this inequality yields a BV regularity result for the unique entropy solution. When this condition fails, we instead obtain a fractional regularity estimate, showing that u belongs to BV^s for some $s \in (0, 1)$, as in the homogeneous case $g \equiv 0$ [6, 7]. We also discuss the much more delicate case of a heterogeneous, space-dependent source term $g = g(x)$, for which such an Oleinik-type inequality is not expected to hold for general integrable functions g [8], and is currently known only in a very special case [9].

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On some typicality and density results for nonsmooth vector fields and the associated continuity equation

Cianfrocca, Francesco; Modena, Stefano

Gran Sasso Science Institute, Italy

Thursday, May 28, 17:15-17:40, M17.12 – 1.005

We are interested in the problem of existence and/or uniqueness of local distributional solutions to the continuity equation associated with a vector field in $L^1_t L^q_x$ where $q \in [1, \infty)$. It is well known that existence of $L^\infty_t L^{q'}_x$ solutions holds for any initial datum in $L^{q'}$ provided the vector field has bounded divergence. Moreover, DiPerna and Lions' theory guarantees uniqueness of such solutions when the vector field belongs to $L^1_t W^{1,q}_x$. Unfortunately, in $L^1_t L^q_x$ only "a few" vector fields satisfy these conditions.Therefore a natural question is: how "typical" is it in $L^1_t L^q_x$ for the existence and/or uniqueness properties to hold without making any further assumptions on the vector field? In this talk we will address this question and we will provide an answer in terms of Baire category. This is a joint work with S. Modena (GSSI).

Supercritical geometric optics and expected regularity for scalar multi-dimensional nonlinear conservation laws

Junca, Stéphane; Cheverry, Christophe; Harb, Rida

LJAD & CNRS, Université Côte d'Azur, France

Thursday, May 28, 17:40-18:05, M17.12 – 1.005

For genuinely nonlinear scalar multi-dimensional conservation laws and $L^\infty(\mathbb{R}^d, \mathbb{R})$ initial data, it is well known that the entropy solution acquires a small fractional regularity $s > 0$ for a space's dimension $d > 1$ [12, 14]. Thanks to weakly nonlinear geometric optics near a constant state $s \leq 1/d$ and the equality is possible [2, 3, 7, 9, 11]. In this talk, in the neighborhood of a non-constant smooth state and for a spatial dimension $d \geq 3$ we show, under a generic condition, that a minimal fractional regularity $s = 1/2$ can be expected.

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P2-1 General Numerical Methods

In-cell discontinuous reconstruction path-conservative methods for conservative systems in primitive variables

Pimentel-García, Ernesto; Mario Ricchiuto, Carlos Parés

University of Málaga, Spain

Monday, May 25, 14:00-14:25, M17.17 – 1.017

We propose a numerical methodology for the treatment of conservative systems formulated exclusively in primitive variables, building upon the framework introduced in [1, 2, 3]. The main motivation is to design numerical methods capable of solving fluid models in primitive variables without altering shock waves. In several applications, the use of primitive variables is not only advantageous but mandatory, since formulations based on conserved variables may produce spurious oscillations near contact discontinuities. However, working solely with primitive variables is known to introduce convergence difficulties in the presence of shocks due to nonconservative products.

These difficulties are addressed here through the identification of a suitable family of paths and the incorporation of in-cell discontinuous reconstructions within a path-conservative framework. In smooth regions and across contact discontinuities, a standard path-conservative method is employed, while in the presence of shocks, an in-cell discontinuous reconstruction is locally activated to ensure the correct resolution of discontinuities. The approach is further extended to achieve high-order accuracy.

Unlike previous hybrid strategies, where primitive variables are used in smooth regions and conservative variables are required near shocks, the present methodology eliminates the need for conservative formulations. Numerical experiments for the Euler multimaterial and shear interface models demonstrate that the proposed scheme converges to the correct weak solution even in the presence of shocks and effectively suppresses spurious oscillations near contact discontinuities, all while working exclusively with primitive variables.

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An energy preserving domain of dependence stabilization method on cut cell meshes

Birke, Gunnar; Engwer, Christian; May, Sandra; Petri, Louis; Ranocha, Hendrik

University of Münster, Germany

Monday, May 25, 14:25-14:50, M17.17 – 1.017

Automated mesh generation for complex geometries can be a costly task. Cartesian cut cell meshes are an attempt to reduce these costs: Objects of interest are embedded in a cartesian mesh and then simply cut out, resulting in so called cut cells along the boundary of the embedded objects. While simple, almost no guarantees can be given about the size and shape of these cut cells, leading to the so called small cell problem. One aspect of it is the severe restriction of the time step size when an explicit Runge-Kutta scheme is used on a cut cell mesh with very small cells.

Over the years many methods have been developed to handle the small cell problem. In this talk we will consider the Domain of Dependence stabilization method. It starts with a typical discontinuous Galerkin scheme and adds to this penalty terms that enlarge the numerical domain of dependence of neighbors of small cut cells via additional flux terms that distribute information across these small cut cells.

We will present an abstract framework for this stabilization method in which we are able to proof energy preservation for linear systems, including at a reflecting wall boundary. Numerical results for the acoustic wave equation support our findings. The framework seems flexible enough to allow for non-linear systems and we will discuss such a possible extension.

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Domain-of-dependence-stabilized cut-cell discretizations of linear kinetic models with summation-by-parts properties

Petri, Louis; Ortleb, Sigrun; Birke, Gunnar; Engwer, Christian; Ranocha, Hendrik

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Monday, May 25, 14:50-15:15, M17.17 – 1.017

In this talk, we present recent results that extend the domain-of-dependence (DoD) stabilization for the discontinuous Galerkin method on cut-cell meshes to linear kinetic models in diffusion scaling using the summation-by-parts (SBP) framework. Numerical methods for these models are challenged by increased stiffness for small scaling parameters and by the necessity of asymptotics preservation regarding a parabolic limit equation. As a prototype model, we consider the telegraph equation in one spatial dimension with periodic boundary conditions, and an asymptotic limit given by the linear heat equation. We provide a general semidiscrete stability result for this model when spatially discretized with arbitrary periodic (upwind) SBP operators and highlight specific implicit-explicit (IMEX) time discretizations for which the fully discrete scheme is asymptotic preserving. Moreover, we sketch how the DoD stabilization with central numerical fluxes leads to periodic SBP operators and how adapting the upwind DoD scheme yields periodic upwind SBP operators. Consequently, a specific application of the DoD stabilization possesses the desired properties mentioned above and thus leads to a stable and asymptotic-preserving scheme for the telegraph equation. Finally, we back our theoretical results with numerical sim-

ulations.

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Coupling a Maxey-Riley solver with Eulerian shallow-water solvers: contrasting finite-volume and E-SPH approaches

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Monday, May 25, 15:15-15:40, M17.17 – 1.017

This study investigates the dynamics of finite-size inertial particles by coupling the Maxey-Riley equation with two distinct Eulerian shallow-water frameworks: a first-order Finite Volume (FV) Godunov-type scheme and a Eulerian Smoothed Particle Hydrodynamics (E-SPH) solver. The primary objective is to evaluate how the numerical discretization of the hyperbolic system influences the Lagrangian statistics of the particles. The computational domain consists of an idealized, high-aspect-ratio compound channel, with obstacles in the floodplain. The longitudinal scale is 10000 times the equivalent particle diameter. The particles have physical properties consistent with common plastics, namely high-density polyethylene. Flow fields are generated under stationary conditions, with the time step constrained by the Courant-Friedrichs-Lewy (CFL) condition of the respective hyperbolic solvers. In the FV approach, the velocity field is piecewise constant, necessitating the use of directional derivatives across cell centers to estimate the fluid accelerations required by the Maxey-Riley terms. Conversely, the E-SPH approach utilizes a meshless kernel-based interpolation to provide an inherently smoother velocity gradient field. Preliminary results indicate non-negligible differences in the statistical properties of the Lagrangian ensembles. The longitudinal diffusion coefficient, derived from the second moment of the particle positions, is larger in the FV-computed fields. Furthermore, the FV simulations exhibit a higher kurtosis compared to the E-SPH results. The current working hypothesis suggests that the cell-jumps of the FV flow field introduces non-physical, localized accelerations at cell interfaces. These numerical artifacts act as stochastic forcing on the Maxey-Riley dynamics, artificially increasing the variance of the particle longitudinal distribution. This work highlights the important role of fluid-field smoothness when modeling the transport of inertial pollutants in shallow-water environments.

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P2-2 General Numerical Methods

Realizability-preserving monolithic convex limiting in continuous Galerkin discretizations of the M_1 model of radiative transfer

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Monday, May 25, 16:00-16:25, M17.17 – 1.017

We discretize the M_1 model of radiative transfer using continuous finite elements and propose a tailor-made monolithic convex limiting (MCL) procedure for enforcing physical realizability. The M_1 system of nonlinear balance laws for the zeroth and first moments of a probability distribution function is derived from the linear Boltzmann equation and equipped with an entropy-based closure for the second moment. To ensure hyperbolicity and physical admissibility, evolving moments must stay in an invariant domain representing a convex set of realizable states. We first construct a low-order method that is provably invariant domain preserving (IDP). Introducing intermediate states that represent spatially averaged exact solutions of homogeneous Riemann problems, we prove that these so-called bar states are realizable in any number of space dimensions. This key auxiliary result enables us to show the IDP property of a fully discrete scheme with a diagonally implicit treatment of reactive terms. To achieve high resolution, we add nonlinear correction terms that are constrained using a two-step MCL algorithm. In the first limiting step, local bounds are imposed on each conserved variable to avoid spurious oscillations and maintain positivity of the scalar-valued zeroth moment (particle density). The second limiting step constrains the magnitude of the vector-valued first moment to be realizable. The flux-corrected finite element scheme is provably IDP. Its ability to prevent nonphysical behavior while attaining high-order accuracy in smooth regions is verified in a series of numerical tests. The developed methodology provides a robust simulation tool for dose calculation in radiotherapy.

A characteristic-based numerical scheme for the optimal control of production–destruction dynamics

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Monday, May 25, 16:25-16:50, M17.17 – 1.017

We introduce a novel framework for the optimal control of production–destruction systems, a class of ordinary differential equations characterized by linear invariants and commonly used to model processes in which components are simultaneously generated and depleted. The control problem is formulated over a finite time horizon and addressed using the Dynamic Programming Principle, which leads to a hyperbolic partial differential equation: the Hamilton–Jacobi–Bellman (HJB) equation [1]. To approximate its viscosity solution, we propose a Modified Patankar Semi-Lagrangian (MPSL) scheme [2]. This numerical method accurately follows characteristic curves, preserving invariant structures and enabling massive parallelization on GPUs. The proposed methodology is validated on two representative applications: an enzyme-catalysed biochemical reaction and a compartmental epidemic model. The results demonstrate that the MPSL scheme outperforms classical approaches, yielding more accurate optimal feedback controls.

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An IMEX-AP scheme for the low-froude shallow water equations with a multigrid-type solver

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Monday, May 25, 16:50-17:15, M17.17 – 1.017

For low Froude number regimes, and analogously for low Mach number regimes, the shallow water and barotropic Euler equations pose a well-known numerical challenge: the scale separation between fast pressure (acoustic) effects and slow advective dynamics imposes severe stability restrictions and may induce non-physical oscillations or excessive diffusion when standard explicit methods are employed. In this work, we develop and analyze an implicit–explicit (IMEX) finite volume scheme with the Asymptotic Preserving (AP) property for the shallow water equations in the low-Froude limit, ensuring that the method remains stable and consistent without the need to explicitly resolve the stiff acoustic scale. The flux decomposition separates convective terms, treated explicitly, from the pressure contribution, which is treated implicitly, thereby preserving the correct dynamical balance in the asymptotic limit and avoiding the typical degeneration of non-AP schemes. A multigrid-type nonlinear solver is employed, and an adaptive residual-based strategy is introduced to improve convergence with ϵ-robust and mesh-independent behavior. The effectiveness of the method is assessed through theoretical results (asymptotic consistency and uniform-in-ϵ stability) and numerical experiments. Additionally, we discuss the potential extension of this approach to an IMEX formulation for the barotropic Euler equations, as part of ongoing work, as part of ongoing work.

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Semi-implicit central scheme for hyperbolic systems of balance laws with relaxed source term

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Monday, May 25, 17:15-17:40, M17.17 – 1.017

Hyperbolic balance laws [2, 3, 5, 4] with stiff source terms pose significant numerical challenges due to severe stability constraints. In this work, we propose a finite volume scheme based on the Nessyahu-Tadmor [1] central framework coupled with a newly developed implicit-explicit (IMEX) time discretisation designed to efficiently treat stiffness while preserving second-order accuracy. Convective fluxes are approximated using the central structure of the Nessyahu-Tadmor scheme, thereby avoiding Riemann solvers and maintaining non-oscillatory resolution near discontinuities. The stiff source term is handled semi-implicitly through a spatial midpoint rule combined with trapezoidal time integration and a backward Taylor expansion. This formulation enlarges the stability region and relaxes time-step restrictions without compromising accuracy.

The method is validated on representative one-dimensional benchmark problems, including the Jin–Xin relaxation system, the shallow-water equations, the Broadwell model, and the Euler equations with heat transfer and stiff friction. To demonstrate robustness and multidimensional capability, the approach is extended to the two-dimensional relaxation systems. Numerical experiments confirm second-order accuracy in both one- and two-dimensional settings and stable behaviour in stiff regimes. Theoretical analysis establishes a large stability region, non-oscillatory behaviour, and asymptotic second-order accuracy as the relaxation parameter approaches zero, ensuring consistency with the limiting equilibrium system. The agreement between theoretical and computational results confirms the reliability and effectiveness of the proposed scheme for hyperbolic balance laws with stiff source terms.

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Convergence of the non-staggered Nessyahu-Tadmor scheme for coupled systems of one-dimensional nonlocal balance laws

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Monday, May 25, 17:40-18:05, M17.17 – 1.017

Systems of balance laws play a fundamental role in modeling and understanding complex physical phenomena, as they describe the interaction of several quantities that evolve over time. A generalized form similar to [1] of a system of N nonlocal balance laws in one spatial dimension can be expressed as follows

$$\partial_t(\boldsymbol{\rho}(t,x)) + \partial_x(F(\rho, \omega * \boldsymbol{\rho})) = S(\boldsymbol{\rho}, \omega * \boldsymbol{\rho}), \quad \boldsymbol{\rho}(t,0) = \boldsymbol{\rho}_0(x) \tag{1}$$

with $t \in \mathbb{R}^+$, $x \in \mathbb{R}$, $\boldsymbol{\rho}(t,x) \in \mathbb{R}^N$, $\omega : \mathbb{R} \to \mathbb{R}^{m,N}$, $(\omega * \boldsymbol{\rho})(t,x) \in \mathbb{R}^m$, $F : \mathbb{R}^N \times \mathbb{R}^m \to \mathbb{R}^N$, $S : \mathbb{R}^N \times \mathbb{R}^m \to \mathbb{R}^N$, with suitable initial data $\rho_0 : \mathbb{R} \to \mathbb{R}^N$. In this work, we assume that the system (1) is only coupled through the nonlocal term or the source, i.e., for $k = 1, \ldots, N$, the equation (1) can be expressed as

$$\partial_t \rho^k(t,x) + \partial_x F_k(\rho^k, \omega * \boldsymbol{\rho})(t,x) = S_k(\boldsymbol{\rho}, \omega * \boldsymbol{\rho})(t,x), \tag{2}$$

with $F_k : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}$ and $S_k : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}$. Here, ω is a suitable kernel function, and $\boldsymbol{R}(t,x) = (\omega * \boldsymbol{\rho})(t,x)$. Central schemes offer several advantages in the numerical approximation of hyperbolic conservation laws. They avoid Riemann solvers and characteristic decomposition, making implementation simpler and suitable for complex systems. For more simplicity, we develop a high-resolution central scheme using non-staggered grids [3] based on the well-known Nessyahu-Tadmor scheme [2] for a general non-local system such as (2). Assuming that the flux and source terms are locally Lipschitz, we can prove that the approximate solutions remain bounded in time by an exponential $L^\infty-$ estimate. Under suitable linearity assumptions on the flux and source terms, the approximate solutions converge weakly-* to a weak solution of the nonlocal balance laws. For nonlinear flux and source terms, we assume additional kernel regularity to control spatial derivatives and obtain a total variation bound. The compact support of kernel provides the necessary estimates and enough compactness to pass to the limit and obtain a weak entropy solution. Because of the specific treatment of the nonlocal convolution term and the imposed regularity assumptions, our NT scheme differs from the classical formulation, which constitutes a key novelty of our approach. We further support our theoretical results with numerical experiments that confirm the convergence and effectiveness of the scheme.

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P2-3 General Numerical Methods

Efficient WENO schemes for nonuniform grids with optimal accuracy

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Tuesday, May 26, 16:00-16:25, M17.17 – 1.017

Non-linear interpolation techniques have been extensively used in several settings such as image processing, data analysis, finance, economy, and overall in solving numerically partial differential equations. In this context, weighted essentially non-oscillatory (WENO) methods have been developed for different problems. The idea of the non-linear methods is to use the data information to avoid the data points where discontinuities or steep gradients appear. The classic WENO combines some polynomial interpolators with a certain order of accuracy to obtain a new interpolator that is more accurate in smooth zones and, at least, as accurate as the original ones when the data present a discontinuity. The construction of the classic WENO method has been formulated for uniform grids and to approximate a determined point of the grid. Recently, the authors introduced in [1] a WENO version for non-uniform grids and used it for the solution of conservation laws and hyperbolic systems in the context of finite volume methods. In this paper, this method is extended and improved, focusing on three aspects. Its efficiency: a more efficient procedure to iteratively compute the reconstruction polynomials for the different types of reconstructions considered in this work is proposed. Its versatility: the designed method can be used in the context of a non-uniform stencil with the reconstruction point located at an arbitrary position. Its order of accuracy: some theoretical results showing that the order of the proposed scheme is optimal on smooth data and as high as possible near discontinuities are presented. Finally, some numerical experiments are performed to illustrate the theoretical results that are presented.

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Interpretation of distributed residual methods with piece-wise defined test functions

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Tuesday, May 26, 16:25-16:50, M17.17 – 1.017

Distributed Residual (DR) schemes are usually employed to solve hyperbolic system in Jacobian form ,

$$\frac{\partial u}{\partial t} + (\mathbf{a}(u) \cdot \nabla)\, u = 0,$$

however their mathematical analysis is, as a rule, done in the case of constant advection:

$$\frac{\partial u}{\partial t} + (\mathbf{a} \cdot \nabla)\, u = 0.$$

Here we specifically interest ourselves in variationnal interpretation for such schemes. Classically, Finite Element (FE) schemes on elements $\mathcal{E}$ of a mesh $\mathcal{M}$ (as well as Finite Volume (FV) schemes on the dual cells $\mathcal{C}$) allow for a DR interpretation. Conversely a variationnal, FE-like interpretation of DR schemes also exists in the literature albeit restricted to the linear, *constant* advection case.
We believe that those results may be quite effortlessly extended to the non-linear case, as the DR schemes themselves hinge usually on quasi-linearized interpretations of the primitive system. This may open interesting perspectives on the adaptation of FE-like techniques on DR schemes, which we shall discuss.

Acknowledgements. This work was undertaken at Électricité De France's Laboratoire National d'Hydraulique et Environnement in the openTelemac project.

Numerical simulations of single-mode Rayleigh–Taylor and Richtmyer–Meshkov instabilities with improved front-tracking/ghost-fluid method

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Tuesday, May 26, 16:50-17:15, M17.17 – 1.017

Theoretical, experimental, and numerical studies are crucial for understanding and predicting the dynamics of hydrodynamic instabilities that occur in a wide range of science and engineering applications. Rayleigh–Taylor instability (RTI) is a hydrodynamical instability driven by an acceleration force applied across a density discontinuity. Richtmyer–Meshkov instability (RMI) is an impulsively accelerated instability that is characterized by the interaction of a shock wave with a fluid interface. When the shock passes through the interface that separates two fluids, the misalignment of density and pressure gradients generates vorticity through the baroclinic mechanism.
Recently, an increasingly accurate and robust front-tracking/ghost-fluid method (FT/GFM) with high-order weighted essentially non-oscillatory (WENO) schemes was introduced for the numerical simulations of RTI [1] and RMI [2]. FT enforces the exact jump conditions at the tracked fluid interface by solving Riemann problems. GFM avoids the unphysical oscillations at the fluid interfaces by separating the domain for each fluid and defining each fluid domain with a ghost-fluid region. WENO ensures that maximum order of accuracy is achieved in the regions away from the interface. These methods are implemented in the FronTier multiphysics software package. For validation, the single-mode RTI experiments of Renoult et al. [3] and RMI shock tube experiments of Collins and Jacobs 2002 [4] for the incident shock strength $M = 1.21$ serve as benchmark tests. In our numerical simulations, two dimensional compressible Euler equations augmented by the mass fraction conservation equation are solved for inviscid flows. The velocity vector fields for the bubble and spike in the linear and nonlinear regimes are investigated and compared with the theory of RTI. The interface dynamics, amplitude growth, and asymmetry on the bubble/spike structures of our simulations show good agreement with the experiments.

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Numerical investigation of nonlinear age-structured tumor cell population model

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Tuesday, May 26, 17:15-17:40, M17.17 – 1.017

We study a nonlinear age-structured model for tumour cell population dynamics that accounts for the proliferating and quiescent phases of the cell cycle. The model is formulated as a system of age-dependent partial differential equations, where the age variable represents the time a cell spends in a given phase. Nonlinear effects arise through bidirectional transitions between the proliferating and quiescent compartments, with transition rates depending on both cell age and the total cell population.
The stability properties of the trivial equilibrium solution are analysed, including both local and global behaviour. For the numerical treatment of the model, a discontinuous Galerkin method is used to discretise the age variable, while time integration is carried out using a fourth-order explicit Runge-Kutta scheme. The accuracy of the numerical method is examined through convergence rate analysis. Numerical simulations are presented to support the results and to demonstrate the influence of model parameters on the qualitative behaviour and long-term dynamics of the tumour cell population.

P2-4 General Numerical Methods

A IMEX-based spectral scheme with adaptive time-stepping for the Vlasov-Poisson system in the quasi-neutral limit

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Thursday, May 28, 14:00-14:25, M17.17 – 1.017

The choice of the time step is important not only for stability, but also for efficiency and consistency with the equations under consideration. In the context of kinetic equations, these aspects are particularly critical. On the one hand, efficiency must be optimized because of the high dimensionality of phase space, which may range from 2 to 6 depending on the problem, leading to a very large computational cost. On the other hand, the simulation of models, especially in Plasma Physics, often involves very fast dynamics that require an appropriate time discretization in order to be accurately resolved. The aim of this talk is to present recent results on time-step selection strategies applied to a numerical scheme for the Vlasov–Poisson equation in the quasi-neutral limit. The scheme is based on a Hermite spectral decomposition in velocity and a finite-difference discretization in space, and was originally proposed in 2025 by Blaustein, Dimarco, Filbet, and Vignal. It relies on a fully implicit, L-stable DIRK time discretization, yielding an Asymptotic-Preserving method in the quasi-neutral limit, at the expense of a significant computational cost per time step. A key issue is that, although the L-stable scheme can capture the oscillating behavior in time of the Vlasov–Poisson system (particularly those of the electric field and current density) in the asymptotic regime, its dissipative nature may eventually damp these oscillations if the time step is not properly chosen. In this work, we instead adopt an IMEX approach, treating the nonlinear part implicitly and the linear part explicitly. This substantially improves the efficiency while maintaining stability, although the Asymptotic-Preserving property is lost. Combined with this semi-implicit treatment, we aim to exploit the accuracy control performed by the time-step selection technique and use it as main ingredient for recovering the fast and ample oscillations in time.
This work was conducted in collaboration with prof. Francis Filbet (University of Toulouse).

A finite volume characteristics scheme for compressible non-equilibrium multiphase flows

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Thursday, May 28, 14:25-14:50, M17.17 – 1.017

Numerical simulation of compressible multiphase flows is used in several sectors. Industries such as aerospace, automotive, nuclear and petrochemicals use these models to design and optimise their systems. Depending on the physical configuration, multiphase flows may involve separated interfaces or dispersed phases. While interfacial flows are often described using equilibrium models, dispersed phases flows require non-equilibrium model and most of the time out of thermal and mechanical equilibrium. Among the available approaches, the Baer-Nunziato model [2] provides a general framework for the simulation of compressible multiphase flows with velocity, pressure, and thermal non-equilibrium between phases.

The numerical approximation of this system remains challenging, mainly because of the presence of non-conservative terms and the loss of classical Rankine-Hugoniot relations. As a consequence, the construction of robust numerical schemes requires particular care. Several numerical

methods based on approximate Riemann solvers have been proposed for the Baer-Nunziato equations (for example [1, 3, 4]). Although these approaches are mathematically solid, they often lead to complex solvers that are computationally expensive, especially when applied to multidimensional configurations or industrial-scale simulations. This motivates the development of alternative strategies that avoid dealing with Riemann problems while maintaining robustness and accuracy.

In this work, we propose an extension of the finite volume characteristic method (FVC), introduced for the Euler equations [5], to the Baer-Nunziato model. The conservative part of the system is treated phase by phase using a predictor-corrector strategy, while the non-conservative terms are discretized using the Abgrall criterion [1]. The proposed method has been implemented and evaluated through a series of numerical tests. The numerical results demonstrate the robustness of the scheme and its ability to accurately resolve contact discontinuities, while remaining computationally efficient.

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Barotropic-baroclinic splitting for multilayer shallow water models with exchanges

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Thursday, May 28, 14:50-15:15, M17.17 – 1.017

Multilayer ocean models (see e.g. [1]) are popular approximations to the 3D Euler and Navier-Stokes equations. Computational cost obviously increases with the number of layers, which is often chosen to be around 50 in ocean simulations. The barotropic-baroclinic splitting is an important strategy used in numerical ocean models to reduce this computational cost (see e.g. [2]).

In the present contribution [3], we focus on the *numerical analysis* of the barotropic-baroclinic splitting in the context of *finite volume schemes*. We reformulate the splitting strategy within the nonlinear multilayer framework using terrain-following coordinates, and present it as an exact operator splitting. The barotropic step captures the evolution of free surface and depth averaged velocity with a well-balanced one-layer shallow water model. The baroclinic step incorporates vertical exchanges between layers and adjusts velocities around their mean vertical value.

Our version of the splitting is numerically robust, i.e. no filters or corrections are needed. The numerical solution inherently observes a discrete maximum principle for the tracer and

hence guarantees non-negative tracer concentrations. In the language of applied mathematics, we prove a discrete entropy inequality. In the language of geophysics, this guarantees dissipation of kinetic and potential, and therefore of total energy. This is *the* key stability property for the class of finite volume schemes under consideration. Last, but not least, the gain in terms of computational cost is large, especially in low Froude simulations.

Currently, this work addresses the constant density case; however, ongoing work extends the barotropic-baroclinic splitting to variable density scenarios and models situations such as coastal upwelling.

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A provably convergent MUSCL-type central scheme for conservation laws with discontinuous flux

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Thursday, May 28, 15:15-15:40, M17.17 – 1.017

We propose a second-order MUSCL-type central scheme for a general class of conservation laws with discontinuous flux and present its convergence analysis. For conservation laws with discontinuous flux, the convergence analysis of second-order schemes based on the minmod limiter has remained open due to the absence of BV (bounded variation) estimates and the non-monotonicity of the scheme. To overcome these challenges, we employ the theory of compensated compactness. Crucial ingredients of the analysis include the derivation of a maximum principle and the proof of $\mathrm{W}^{-1,2}_{\mathrm{loc}}$-compactness for suitable sequences constructed from the approximate solutions. Furthermore, a mesh-dependent modification of the slope limiter, together with an estimate weaker than the classical BV bound, is used to extend a classical entropy convergence framework to the setting of conservation laws with discontinuous flux. Numerical experiments demonstrate the improved accuracy of the proposed scheme compared with a first-order method.

P2-5 General Numerical Methods

Supersonic reacting jet flows from three-dimensional divergent conical nozzles

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Thursday, May 28, 16:00-16:25, M17.17 – 1.017

In this talk, the existence of global supersonic reacting jet flows from three-dimen-sional (3D) divergent conical nozzles is shown. These type of flow structures are frequently appear in aero engines. In order to construct such a supersonic reacting jet flow, we consider a boundary value problem for the 3D steady Zeldovich-von Neumann-Döring (ZND) combustion equations in gas dynamics. Global existence of piecewise smooth nonisentropic rotational supersonic solution to the boundary value problem is obtained by the method of characteristics. One of the main difficulties for the global existence is that a priori C^1 norm estimates of the solution is hard to obtain. The other difficulty is that there exists a multiple-valued singularity at the exit edge of the nozzle. This may also make it difficult to establish the a priori C^1 estimates of the solution. We derive a group of characteristic decompositions for the 3D steady ZND combustion equations with cylindrical symmetry. Using these decompositions, we establish the a priori C^1 estimate. The supersonic reacting jet flows constructed in the paper may be also used as background solutions for some other general supersonic jet problems.

This is a joint work with Prof. Geng Lai.

Experimental and non-hydrostatic Saint-Venant-Exner modeling of dike breach by overtopping

Moya, Jorge; Jónatas, Ricardo J. L.; Ferreira, Rui M. L.; Amaral, Silvia; Escalante, C.; Fernández-Nieto, Enrique D.

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Thursday, May 28, 16:25-16:50, M17.17 – 1.017

This study presents an integrated experimental and numerical investigation of river dike breach processes, combining large-scale physical modeling with depth-averaged, non-hydrostatic shallow-water flow and sediment transport simulations. The experimental facility, located at LNEC, reproduces realistic riverine hydraulic conditions and controlled dike failures, enabling systematic analysis of breach initiation and development under varying flow and soil properties. A series of breach experiments was conducted using erodible river dikes constructed from compacted soils with different grain size distributions and moisture contents. High-resolution measurement systems provided continuous monitoring of river water levels, breach geometry evolution, and flow velocities during overtopping-driven failure. The experimental data were used to calibrate and validate the numerical model, which accounts for non-equilibrium sediment transport and pressure effects relevant to rapidly varying river flows. Model performance was assessed by comparing simulated and observed breach widening and deepening rates under river overtopping conditions. The combined experimental–numerical approach enhances the understanding of river dike breach dynamics and contributes to improved modeling capabilities for problems related to rivers restoration.

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2021.07337.BD, by Junta de Andalucía research project ProyExcel_00525, by the European Union – NextGenerationEU program, and by grants PID2022-137637NB-C21 and PID2022-137637NB-C22 funded by MCIN/AEI/10.13039/501100011033 and "ERDF A way of making Europe".

Combined compressible-incompressible multiphase modelling approach of solid propellant ignition and combustion using an extended discontinous Galerkin method

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Thursday, May 28, 16:50-17:15, M17.17 – 1.017

High-fidelity numerical models for the ignition and combustion of granular solid propellant beds in confined chambers are crucial for the design and safety assessment of high-pressure gas generation devices and impulsive actuators in various engineering applications, where realistic configurations involve complex three-dimensional packings of reactive grains.
From the perspective of partial differential equations and numerical analysis, these systems result in strongly transient, compressible multiphase flows through densely packed threedimensional grain configurations. At the continuum scale, this flow can be described by nonlinear hyperbolic conservation laws for a reactive gas phase coupled to an effectively incompressible particulate phase via moving reaction interfaces.
Existing engineering models typically rely on spatially averaged descriptions with pressure-based burning laws or simplified two-phase continuum formulations. Furthermore, many multidimensional simulations prescribe homogeneous surface regression or treat the granular bed as a single continuum, without modelling or discretizing the fully coupled compressible-incompressible gas-solid initial-boundary-value problem with sharp interfaces. As a consequence, these approaches cannot resolve detailed gas-solid jump conditions and the associated multimaterial Riemann problems. They also cannot capture strongly heterogeneous local burning, the resulting shocklike pressure waves, and other interfacedriven phenomena observed in realistic systems, while maintaining highorder accuracy and conservation.
The objective of this work is to develop a mathematically consistent high-order framework that addresses these limitations. We formulate a multidimensional combined compressible-incompressible model with moving gas-solid interfaces for multiphase solid propellant ignition and combustion, and discretize it using an extended discontinuous Galerkin (XDG) method. To this end, we represent the gas-solid interface by a levelset function on a fixed mesh, construct enriched discontinuous Galerkin (DG) approximation spaces within the XDG framework that allow discontinuities aligned with the evolving interface, and design conservative interface numerical fluxes and coupling conditions for the mixed hyperbolic problem, with particular emphasis on consistent jump conditions and global conservation properties.
In this work, we demonstrate, on a prototypical multidimensional configuration, that the resulting XDG scheme is stable and approximately conservative. It is able to resolve heterogeneous ignition fronts, strong pressure waves, and complex interface motion within a unified high-order hyperbolic setting that is directly relevant to the analysis and numerical approximation of nonlinear conservation laws for solid propellant ignition and combustion.

Smooth perturbations of diagonally implicit Runge–Kutta methods

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Thursday, May 28, 17:15-17:40, M17.17 – 1.017

This research expands upon the mixed accuracy framework for Runge–Kutta methods. Our work develops an accuracy and stability analysis to design methods that strategically replace original operators with lower-accuracy versions to reduce computational costs. Specifically, we focus on designing novel methods that maintain high order for smooth perturbations by satisfying additional local consistency conditions. The paper concludes with a numerical study verifying the performance, accuracy, and stability of these perturbed DIRK methods.

P3-1 Blowup and Stability

Dynamic stability of Nash equilibria for a conservation law model of traffic flow

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Monday, May 25, 14:00-14:25, M17.21 – 2.005

We consider a traffic model based on the LWR conservation law where drivers select departure times to minimize individual travel costs. In this setting, the existence of a unique departure rate yielding a Nash equilibrium was proved in [1]. We show that, for generic departure and arrival cost functions, this equilibrium solution contains shock waves. Following [2], we then study a dynamics modeling how drivers adjust their day-by-day departure times, in order to reduce their own total cost. When this dynamics is described by an integro-differential equation, we prove global existence and uniqueness of the evolution of the departure rate, as time increases. Numerical simulations indicate the stability/instability of the Nash equilibrium, in different regimes [3].

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Analytical and numerical construction of tangent vectors to multi-dimensional scalar conservation laws.

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Monday, May 25, 14:25-14:50, M17.21 – 2.005

We investigate the effect of perturbations in the initial data on the solution of a multi-dimensional scalar conservation law $\partial_t u + \nabla \cdot f(u) = 0, \quad u(0,\cdot) = u_0(\cdot)$.
For hyperbolic conservation laws, this is challenging because both the continuous parts of the solution and the shock positions are influenced by the initial perturbations. Consequently, the limit of the difference quotient does not, in general, belong to $L^1(\mathbb{R}^n)$. This was proven in [1] in the one-dimensional case. Therefore, for one-dimensional systems, Bressan and Marson developed a calculus for first-order variations of the continuous parts of the solution and the shock positions. Several applications of this calculus can be found in ongoing research, e.g., in combination with algorithmic differentiation in [2].
We extend this calculus to higher spatial dimensions, taking the geometry of the discontinuities into account. Now, shock curves or surfaces instead of positions must be considered. To deal with this, we perform the approximation of the perturbed solution by shifts along the normals to the shock surfaces. In particular, we include variations of the normal when perturbations are introduced. In this way, we derive evolution equations for the tangent vector (v, r), which consists of the variations of the continuous parts of the solution v and the shock curves r.
First work in this regard was performed in [3], where the authors consider discontinuity curves that split the bounded domain into two parts and use the tangent vector to solve an optimization

problem. We develop a more general calculus that also applies to discontinuity curves not attached to the domain boundary. Moreover, we are interested in computing approximations of perturbed solutions. To verify that the approximation represents a first-order variation, we compare it with the numerically computed perturbed solution.
Several numerical difficulties arise. For example, a fine grid is needed to adequately resolve small perturbations. For the sake of efficiency and to prevent oscillations, we use adaptive grids for the non-perturbed solution u, the tangent vector components v and r, and for constructing the approximate solution. Another difficulty is solving the linearized conservation law for v only away from the discontinuity. This requires particular care, especially when the discontinuity surface does not touch the domain boundary.

Acknowledgements. This work is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 320021702/GRK2326 – Energy, Entropy, and Dissipative Dynamics (EDDy)

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Gas flow of mixtures on networks: existence of steady states with non-constant compressibility factors

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Monday, May 25, 14:50-15:15, M17.21 – 2.005

Climate change compels for a transition away from fossil fuels, positioning hydrogen as a promising alternative, with hydrogen-natural gas mixtures being a viable, interim strategy to reduce greenhouse gas emissions. Although this leads to a growing interest in modeling and optimizing the gas flow of mixtures on networks, only limited theoretical results are available. Thus, it is important to gain a mathematical understanding of the underlying flow dynamics of gas mixtures to ensure safe and efficient operation of the network.

We present a mathematical framework to describe the gas flow of mixtures on networks. The model is based on the physical principles of the isothermal Euler equations and the mixing of incoming flow at nodes. Another key component of the model is the so-called *pressure law* or *equation of state*, which links the pressure p with the gas density ρ and the gas composition η:

$$p(\eta, \rho) = \frac{RT}{M(\eta)} Z(\eta, p)\rho,$$

where R denotes the universal gas constant, T the gas temperature, $M(\eta)$ the molar mass of the mixture, and Z the compressibility factor. As the extension of well-established compressibility models for natural gas to gas mixtures is often not unique, we impose only mild assumptions on

the compressibility factor Z. Under these assumptions, we prove the existence (and uniqueness) of solutions to the steady-state model on tree-shaped networks and networks containing a cycle.

The key idea of the proof is to cut an edge of the cycle and then determine boundary data at the newly created nodes such that a solution on the resulting tree-shaped network is also a solution on the original one. There are two major challenges that we need to overcome: (a) gas mixtures introduce discontinuities into the model, since the gas composition depends discontinuously on the flow, and (b) the steady-state Euler equations reduce to an ordinary differential equation (ODE) for the pressure profile along a pipe, which in general lacks a closed-form solution. Together, these properties make theoretical analysis more intricate, as the existence result relies on the continuous dependence on the boundary data.

Finally, we underline our theoretical results through numerical experiments and illustrate the applicability of our proof to networks with more complex topologies, such as those containing multiple cycles. Further, we showcase the influence of the compressibility factor on the steady-state solution.

[1] S. Göttlich, M. Schuster, and A. Ulke. On the Existence of Steady States for Blended Gas Flow with Non-Constant Compressibility Factor on Networks., arXiv:2602:08481, 2026.

[2] A. Ulke, M. Schuster, and S. Göttlich. Steady state blended gas flow on networks: Existence and uniqueness of solutions. *Networks and Heterogeneous Media*, 20(3):903–937, 2025.

Time periodicity of global classical solution to radially symmetric unsteady flows with nonzero angular velocity in an annulus

Liu, Jianli

Shanghai University, China

Monday, May 25, 15:15-15:40, M17.21 – 2.005

For steady inviscid compressible flows with radial symmetry in an annulus, Weng et al. have demonstrated the global existence of steady solutions [J. Differential Equations., 286:433454, 2021]. We consider the global stability of steady supersonic solution to radial symmetric flows with nonzero angular velocity in an annulus. Furthermore, we can also get the temporal periodicity of classical solutions under the assumption of time-periodic boundary conditions. This work is jointed with Pu Gao and Wancheng Sheng.

P3-2 Blowup and Stability

Repeated blowups: fluids and Yang-Mills

Glimm, James

Stony Brook University, USA

Tuesday, May 26, 14:00-14:25, M17.21 – 2.005

We present a weak solution of the incompressible Navier-Stokes equation which blows up in finite time. We also announce a solution of the quantum Yang-Mills equation with a mass gap at low temperature. Some indication of the ideas and methods involved in the proofs will be presented. Analyticity, the maximization of production of entropy, an analysis of the terms not summed over repeated indices (NSRI) and the Leray-Hopf existence of weak Navier-Stokes solutions are important tools. Kolmogorov62 is used to show that the weak Navier-Stokes solutions lack a single derivative to be solutions in the function space $\mathcal{H}$.

The Yang-Mills phase transition is announced but not presented in detail in this report. Additional steps for the Yang-Mills phase transition include renormalized perturbation theory, truncated to all finite orders, the Borchers-Yngvason theory of weak clustering and the Dyson equation.

The main thrust of this presentation will be to understand the mechanisms and causes of blowup and to embed this understanding in a theory for a second order phase transition.

Stability of planar rarefaction waves in the vanishing dissipation limit of the Navier–Stokes–Fourier system

Neves, Wladimir; Feireisl, Eduard

Federal University of Rio de Janeiro, Brazil

Tuesday, May 26, 14:25-14:50, M17.21 – 2.005

In this talk, we consider the vanishing dissipation limit of the compressible Navier–Stokes–Fourier system, where the initial data approach a profile generating a planar rarefaction wave for the limit Euler system. We show that the associated weak solutions converge unconditionally to the planar rarefaction wave strongly in the energy norm.

Density of "wild" initial data for the isentropic Euler equations considering global-in-time admissible weak solutions

Markfelder, Simon; Boutros, Daniel W.

University of Konstanz, Germany

Tuesday, May 26, 14:50-15:15, M17.21 – 2.005

Using the technique of convex integration, it was proved in the groundbreaking work [3] that there exist initial data to the incompressible as well as to the compressible Euler equations in multiple space dimensions, which allow for infinitely many weak solutions. One can even achieve that all of these solutions are *admissible* in the usual sense that energy / entropy conditions hold. In the sequel, such initial data which give rise to infinitely many admissible weak solutions, will be referred to as *wild initial data*.

The natural question of how generic wild data are, has been answered in [4] in the context of the incompressible Euler equations, by showing that there exists a *dense* set of wild initial

data. Here, the solutions only satisfy a weak version of the energy inequality, namely the energy is bounded by the initial energy for all times.

In this talk, we are interested in the isentropic compressible Euler system. For the latter equations an analogue of [4] was shown in [1], i.e. there exists a dense set of wild data, where the energy of the solutions is at any time bounded by the initial energy. However, in the context of the isentropic Euler equations, one usually asks for the local energy inequality to hold, which is much stronger than merely requiring the energy to be bounded by the initial energy at all times. In [2] a dense set of wild initial data was found, where the solutions now satisfy the local energy inequality as desired. Yet, the solutions in [2] exist only locally-in-time.

In this talk, we present another result regarding density of wild initial data for the isentropic Euler equations. In particular, we consider global-in-time solutions, which are admissible in the sense that the local energy inequality holds. The proof requires a convex integration approach which includes the energy inequality into the differential inclusion, together with a clever construction of subsolutions.

Acknowledgements. S.M. acknowledges financial support from the Alexander von Humboldt foundation and also from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) within SPP 2410, project number 525935467.

[1] R. M. Chen, A. Vasseur and C. Yu. Global ill-posedness for a dense set of initial data to the isentropic system of gas dynamics. *Adv. Math.*, 393: Paper No. 108057, 2021.

[2] E. Chiodaroli and E. Feireisl. On the density of "wild" initial data for the barotropic Euler system. *Ann. Mat. Pura Appl. (4)*, 203(4): 1809–1817, 2024.

[3] C. De Lellis and L. Székelyhidi Jr.. On admissibility criteria for weak solutions of the Euler equations. *Arch. Ration. Mech. Anal.*, 195(1): 225–260, 2010.

[4] L. Székelyhidi Jr. and E. Wiedemann. Young measures generated by ideal incompressible fluid flows. *Arch. Ration. Mech. Anal.*, 206(1): 333–366, 2012.

Weak-BV stability for Euler with boundaries

Meng, Jiayun; Kang, Moon-Jin; Oh, HyeonSeop; Vasseur, Alexis

The University of Texas at Austin, USA

Tuesday, May 26, 15:15-15:40, M17.21 – 2.005

Dafermos and DiPerna have introduced the Weak-Strong Principle for the stability of Lipschitz solutions to conservation laws. The a-contraction theory is a recently developed energy-based method, which extends this principle to the stability of small BV solutions to the Euler equations. In this talk, we will explain how stability results, originally obtained in the whole-space setting, can be extended to boundary value problems. More precisely, we prove Weak-BV stability for Euler weak solutions satisfying the strong trace property in all subsonic boundary configurations: impermeable, inflow, and outflow. In addition, we establish Inviscid limits-BV stability from barotropic Navier-Stokes to Euler for the impermeable and outflow cases. The Weak-BV stability result is joint work with Moon-Jin Kang, HyeonSeop Oh, and Alexis Vasseur, and the Inviscid-BV stability result is joint with Alexis Vasseur.

P3-3 Blowup and Stability

Development of implosions of solutions of the three-dimensional degenerate compressible Navier-Stokes equations

Liu, Lihui; Chen, Gui-Qiang G; Zhu, Shengguo

Shanghai Jiao Tong University, P. R. China

Tuesday, May 26, 16:00-16:25, M17.21 – 2.005

A fundamental open problem in the theory of the multidimensional compressible Navier–Stokes equations is whether smooth solutions can develop singularities—such as cavitation or implosion—in finite time. In the case of constant viscosity coefficients, recent remarkable results show that there exist some smooth initial data for which the corresponding smooth solutions of the barotropic flow undergo finite-time implosion at the origin in three spatial dimensions, with the density ρ blowing up to infinity. On the other hand, when the viscosity coefficients depend linearly on ρ (as in the shallow water equations), it has also been established recently that, for general large spherically symmetric initial data (even allowing vacuum states), the solutions of the barotropic flow remain globally regular in two and three spatial dimensions. These observations indicate that the behavior of the multi-dimensional solutions depends sensitively on the structures of the viscosity coefficients, thereby complicating the analysis of singularity formation. In this paper, we analyze the case of nonlinear density-dependent viscosity coefficients (*i.e.*, ρ^δ with an exponent $\delta > 0$). We identify a value $\delta^*(\gamma) < \frac{1}{2}$, depending on the adiabatic exponent γ, such that, for every $0 < \delta < \delta^*(\gamma)$, there exists a class of smooth initial data for which the corresponding smooth solutions of the barotropic flow implode (with infinite density) in finite time at the origin in three spatial dimensions. Moreover, we assume that the initial density is strictly positive, in order to rule out the possibility that the corresponding implosion is an artifact of the vacuum. The key point is to prove that, in this regime, the degenerate viscous terms are not strong enough to suppress the convective mechanism driving the implosion. However, verifying this issue is challenging due to fundamental difficulties arising from the degenerate structure. Specifically, for flows with constant viscosity coefficients the dissipative effect in the momentum equation remains spatially uniform. In contrast, in our setting, the dissipative effect increases significantly in regions of large density. Therefore, one would expect the viscous terms to suppress any tendency toward implosion. To overcome this obstacle, we first provide a pointwise estimate of ρ by a region segmentation method, and then establish the spatial decay estimates for the gradient of the velocity via carefully designed weighted estimates for high-order derivatives and interpolation inequalities. The corresponding decay rate is fast enough to absorb the singularity of ρ and then leads to a uniform-in-time control of the viscous terms.

The estimate of lifespan for ideal incompressible MHD systems

Li, Mengni; Yuan, Hongwei

Southeast University, China & University of Würzburg, Germany

Tuesday, May 26, 16:25-16:50, M17.21 – 2.005

This talk concerns the lifespan estimate for three-dimensional ideal incompressible MHD systems. By introducing $L^{3/2}$-type energy, we lower the regularity assumption for the initial data as well as the order of the highest derivatives involved in the total weighted energy estimate. The new weights are symmetric, unified, and compatible with both L^2-type energy and $L^{3/2}$-type energy. This is based on joint work with Hongwei Yuan.

Stability analysis for the generalized viscous Burgers' equation with time delay

Ueda, Yoshihiro

Kobe University, Japan

Tuesday, May 26, 16:50-17:15, M17.21 – 2.005

In this talk, we consider the stability of the non-zero equilibrium state for the generalized viscous Burgers' equation with a delay effect. The linear stability is analyzed by using the characteristic equation of the corresponding eigenvalue problem. If our equation does not have a delay effect, the characteristic equation is given by a polynomial equation. On the other hand, if our equation has a delay effect, the characteristic equation becomes a transcendental equation, and it is not easy to analyze it. In this situation, we apply the useful known result concerned with the characteristic equation for the ordinary delay differential equations and try to get the sharp stability condition for our equation. In addition, the nonlinear stability will be studied in the second half of the talk. Furthermore, if there is time remaining, we will discuss the stability of nonlinear waves.

Similarity solutions and nonlinear wave interaction in a new hyperbolic model of two-phase thin film flow of perfectly soluble antisurfactant

Jangid, Hari Om; Raja Sekhar, T.

Indian Institute of Technology Kharagpur, India

Tuesday, May 26, 17:15-17:40, M17.21 – 2.005

This work presents symmetry analysis and nonlinear wave interaction in a novel hyperbolic model of two-layer thin film flow of two immiscible, incompressible and viscous fluids in the presence of a perfectly soluble anti-surfactant, described by the following system of equations

$$\begin{cases} E_1: & \dfrac{\partial f}{\partial t} + fb\dfrac{\partial f}{\partial x} + \dfrac{f^2}{2}\dfrac{\partial b}{\partial x} = 0, \\ E_2: & \dfrac{\partial b}{\partial t} + fb\dfrac{\partial b}{\partial x} + \dfrac{b^2}{2}\dfrac{\partial f}{\partial x} = 0, \\ E_3: & \dfrac{\partial g}{\partial t} + gq\dfrac{\partial g}{\partial x} + \dfrac{g^2}{2}\dfrac{\partial q}{\partial x} + bg\dfrac{\partial f}{\partial x} + fg\dfrac{\partial b}{\partial x} + fb\dfrac{\partial g}{\partial x} = 0, \\ E_4: & \dfrac{\partial q}{\partial t} + gq\dfrac{\partial q}{\partial x} + \dfrac{q^2}{2}\dfrac{\partial g}{\partial x} + bq\dfrac{\partial f}{\partial x} + fq\dfrac{\partial b}{\partial x} + fb\dfrac{\partial q}{\partial x} = 0, \qquad x \in \mathbb{R},\ t > 0, \end{cases} \tag{3}$$

where t is time, x is space coordinate, $h = h(x,t)$ and $g = g(x,t)$ correspond to the film thickness for phase-1 and phase-2, respectively while $b = b(x,t)$ and $q = q(x,t)$ represent the concentration gradient for phase-1 and phase-2, respectively. We determine all the groups of transformations for which the set of solutions of the PDE system (3) remains invariant and classify these groups into inequivalent group classes through the push-forward action. Then by leveraging these classes, exact solutions are derived for the PDE system (3). Additionally, the evolutionary behavior of the film thickness and concentration gradient is characterized by geometric representation of the solutions. Finally, utilizing one of the obtained solutions, the propagation of characteristic shock, weak discontinuity behavior and the corresponding collision between the characteristic shock and the weak discontinuity are discussed.

Asymptotic stability of steady transonic flows in nozzles

Ohnawa, Masashi

Tokyo University of Marine Science and Technology, Japan

Tuesday, May 26, 17:40-18:05, M17.21 – 2.005

In this talk, we show the asymptotic stability of steady transonic flows in nozzles of finite length. The most basic model that describes how fluid machines such as jet engines accelerate a subsonic gas flow to supersonic speeds is the quasi-one-dimensional compressible Euler equations:

$$(A\rho)_t + (Am)_x = 0, \tag{4a}$$

$$(Am)_t + \left(A(\frac{m^2}{\rho} + p(\rho))\right)_x = A_x p(\rho). \tag{4b}$$

The domain $I = (x_1, x_2) \ni 0$ is a finite interval in $\mathbb{R}$, and $A = A(x)$ is the cross-sectional area of the nozzle. The unknown variables $\rho = \rho(t, x)$ and $m = m(t, x)$ represent density and momentum, respectively, and the pressure is assumed to obey the barotropic model.

For the steady problem associated with (4) under the boundary conditions of

$$(A\bar{m})(x_1) = Q(> 0) \quad \text{and} \quad \bar{\rho}(x_2) = \rho_+, \tag{5}$$

we consider three types of stationary solutions $(\bar{\rho}, \bar{m})$:

1. The flow is entirely subsonic.
2. A subsonic flow is continuously accelerated to become a supersonic state.
3. The same as the second type, but afterwards returns to a subsonic state discontinuously.

We claim that all of them are asymptotically stable to small perturbations without any limitation on the sizes of Q and $|A'(x)|$. For the unsteady problem (4) around stationary solutions of these types, one boundary condition at $x = x_1$ is prescribed by

$$A(x_1)m(t, x_1) = Q. \tag{6}$$

For Types 1 and 3, where the flow is subsonic at the exit, another boundary condition at $x = x_2$ is given by

$$\rho(t, x_2) = \rho_+. \tag{7}$$

Because of space constraints, we restrict ourselves to stating the theorem on the asymptotic stability of the stationary solution of Type 2.

Theorem Assume $A \in C^4(\bar{I})$ with $A_x(0) = 0$ and $A_{xx}(0) > 0$. Furthermore, we suppose that the stationary solution $(\bar{\rho}, \bar{m}) \in C^3(\bar{I}) \times C^4(\bar{I})$ to (4) and (5) exists, and that the Mach number $\bar{M}(x)$ satisfies

$$\inf_{x \in \bar{I}} \bar{\rho}(x) > 0, \qquad \begin{cases} \bar{M}(x) < 1 & \text{for } x \in [x_1, 0), \\ \bar{M}(x) = 1 & \text{for } x = 0, \\ \bar{M}(x) > 1 & \text{for } x \in (0, x_2]. \end{cases}$$

Then the stationary solution is asymptotically stable if the initial perturbation $(\rho^0 - \bar{\rho}, m^0 - \bar{m}) \in H^2(I) \times H^2(I)$ is sufficiently small.

Acknowledgements. This study is based on the joint work with Masahiro Suzuki(Nagoya Institute of Technology). This research has been supported by JSPS Grants-in-Aid for Scientific Research, Grant Number 17K05313, 21K03305, and 25K07079.

P4-1 Probabilistic Models

The zero-noise limit for hyperbolic conservation laws

Fjordholm, Ulrik Skre

University of Oslo, Norway

Monday, May 25, 16:00-16:25, M17.21 – 2.005

Over the past two decades, the phenomenon of *regularization by noise* has received an enormous amount of attention. Put simply, this is a general phenomenon in which a stochastic differential equation might exhibit better properties — e.g., existence or uniqueness of solutions, or increased regularity — compared to its deterministic counterpart. An early example is the seminal work of Flandoli, Gubinelli & Priola (Inventiones Mathematicae, 2010), where the authors prove that a stochastically perturbed transport equation is well-posed even for very irregular (Hölder-continuous) velocity fields.

The purpose of this work is threefold. First, we design a novel stochastical perturbation of general scalar conservation laws, and prove that this SPDE is well-posed, *without imposing any entropy conditions*. Second, we show that there is sufficient compactness to pass the noise parameter to zero. Third, we use a novel stability property of scalar conservation laws to prove that the zero-noise limit is in fact the entropy solution of the deterministic problem. In this way, the addition of (a vanishing amount of) noise acts as a selection mechanism, which coincides with the standard entropy conditions.

This is joint work with Ola Mæhlen (Université Paris-Saclay) and Magnus Ch. Ørke (University of Oslo).

On the dynamics of the 1D stochastic Quantum-Navier-Stokes equations

Pescatore, Lorenzo

University of L'Aquila, Italy

Monday, May 25, 16:25-16:50, M17.21 – 2.005

We consider a stochastically forced compressible viscous fluid with capillarity. For $x \in \mathbb{T}$, the one-dimensional flat torus, the dynamics is described by the stochastic Quantum-Navier-Stokes equations:

$$\begin{cases} \partial_t \rho + \partial_x(\rho u) = 0 \\ \mathrm{d}(\rho u) + \big[\partial_x(\rho u^2) + \partial_x p(\rho)\big]\mathrm{d}t = \big[\partial_x(\mu(\rho)\partial_x u) + \rho\partial_x\Big(\dfrac{\partial_{xx}\sqrt{\rho}}{\sqrt{\rho}}\Big)\big]\mathrm{d}t + \rho\mathbb{F}(\rho,u)\mathrm{d}W \end{cases} \tag{8}$$

$$p(\rho) = \rho^\gamma, \quad \gamma > 1, \qquad \mu(\rho) = \rho^\alpha, \quad \alpha \geq 0,$$

endowed with random initial data and periodic boundary conditions. The process W is a cylindrical Wiener process and the related stochastic integral has to be understood in the Itô sense. We prove well-posedness results for system (8) in the framework of strong and weak solutions in both PDEs and Probability sense. The analysis is performed for a wide class of density dependent viscosity coefficients $\mu(\rho) = \rho^\alpha$, $\alpha \geq 0$. In particular, we first prove a general local existence result in the class of strong pathwise solutions. Then, by means of Bresch-Desjardins entropy, higher order regularity estimates and a continuation argument we prove that for $0 \leq \alpha \leq 1/2$ the density never vanishes and thus that local strong solutions are indeed global. In the range of the viscosity parameter $1/2 < \alpha \leq 1$, vacuum regions may appear and thus only existence

of weak solutions is expected. In particular, we prove the existence of weak dissipative martingale solutions by using an approximating system which provides extra dissipation properties and the convergence is based on an appropriate truncation of the velocity field in the momentum equation and a stochastic compactness argument. As a byproduct, our analysis covers also the deterministic case. This is a joint work with Donatella Donatelli and Stefano Spirito (University of L'Aquila).

[1] D. Donatelli, L. Pescatore and S. Spirito, Global regularity for the one-dimensional stochastic Quantum-Navier-Stokes equations. *Preprint, available at arXiv:2401.10064*, (2024).

[2] D. Donatelli, L. Pescatore and S. Spirito, Weak martingale solutions to the stochastic 1D Quantum-Navier-Stokes equations. *Preprint, available at arXiv:2412.10875*, (2024).

[3] L. Pescatore, *The stochastic compressible Navier-Stokes-Korteweg system: The one-dimensional case*, Ph.D. Thesis, University of L'Aquila, (2025).

Probabilistically strong solutions to stochastic Euler equations

Lasarzik, Robert; Gess, Benjamin

Weierstrass Institute for Applied Analysis and Stochastics, Germany

Monday, May 25, 16:50-17:15, M17.21 – 2.005

In this talk, we study the stochastic incompressible Navier–Stokes and Euler equations with both additive and transport noise. For these systems, we establish the existence of probabilistically strong solutions. In particular, we prove the existence of probabilistically strong analytically measure-valued solutions to the stochastic incompressible Navier–Stokes equations and show that, in the vanishing viscosity limit, they converge to probabilistically strong solutions of the stochastic incompressible Euler equations. To achieve this, we introduce the notion of energy-variational solutions in the stochastic setting. This framework allows us to treat the nonlinearities without changing the underlying probability space and provides a natural way to identify the vanishing viscosity limit in the energy-variational sense. Finally, we show that the energy-variational formulation enables the reconstruction of a dissipative weak formulation for the stochastic incompressible Euler equations, incorporating a measure-valued Reynolds defect.

This is a joint work with Benjamin Gess.

Mixing at the Batchelor scale for white-in-time flows

Chemnitz, Dennis; Chemnitz, Robin

ETH Zürich, Switzerland

Monday, May 25, 17:15-17:40, M17.21 – 2.005

Understanding the nature of turbulence occurring in three-dimensional high Reynolds number fluid flows is one of the major open problems in continuum mechanics. In particular, the Kolmogorov -5/3 scaling, which predicts the energy distribution of a turbulent fluid flow over Fourier modes, is wide open. A related, but mathematically more tractable, problem concerns the Fourier mass distribution of the solution to an advection-diffusion equation for which the advection vector field is given by a turbulent fluid flow. Such an equation models the evolution of passive scalars, such as, for example, the concentration of particles dissolved in the fluid or the temperature of the fluid.

In recent years, there has been tremendous progress in understanding the mixing of passive scalars advected by random velocity fields in the Batchelor regime. In particular, it has been shown that transport by solutions of the Navier–Stokes equation enhances the dissipation of passive scalars yielding a uniform-in-diffusivity lower bound for the dissipation rate. While this is expected to be optimal, the corresponding upper bound remains open, a problem which is known as the Batchelor-scale conjecture.

In this talk, we consider the mixing properties of solutions to the advection-diffusion equation of a white-in-time velocity field on the 2-dimensional torus with four forced modes. As the diffusivity parameter goes to zero, we show that the almost-sure exponential dissipation rate stays bounded from above. To our knowledge, this constitutes the first example of a velocity field for which the Batchelor scale conjecture can be verified.

Structure-preserving local discontinuous Galerkin methods for SPDEs with gradient noise

Christiansen, Thomas; Karlsen, Kenneth H.

University of Oslo, Norway

Monday, May 25, 17:40-18:05, M17.21 – 2.005

In this talk, based on joint work with Kenneth H. Karlsen (UiO), we discuss a framework, based on the local discontinuous Galerkin (LDG) method, for discretizing linear and nonlinear transport equations perturbed by gradient noise in Stratonovich form. This noise has been prompted as a natural way of modeling how small-scale randomness influence macroscopic transport phenomena. In the most general case, we can treat stochastic-convection diffusion equations of the form

$$du + \operatorname{div}_x f(u)\,dt + \sum_{\ell\in L} \operatorname{div}_x\big(\sigma_\ell(x) g_\ell(u)\big) \circ dW_t^\ell = \operatorname{div}_x\big(a(u)\nabla u\big)\,dt,$$

where the process $u : \Omega \times [0,T] \times \mathbb{R}^d \to \mathbb{R}$ is the sought unknown, $f : \mathbb{R} \to \mathbb{R}^d$ is the convection field, L is a finite index set, $\sigma_\ell : \mathbb{R}^d \to \mathbb{R}^d$ are driving noise components, $g_\ell : \mathbb{R} \to \mathbb{R}$ are nonlinear functions of the unknown, $\{W^\ell\}_{\ell\in L}$ are mutually independent standard Brownian motions, and $a : \mathbb{R} \to \mathbb{R}^{d\times d}$ is a symmetric, positive semi-definite matrix, representing anisotropic diffusion.

When deriving the LDG-scheme, we convert the equation to its Itô-formulation and the crux is how one discretizes the corresponding Itô–Stratonovich correction term, which is a possibly degenerate second order elliptic operator, in such a way that the hyperbolic-parabolic character of the governing equation is preserved. The LDG-framework is shown to be well suited for this task and the proposed schemes are shown to exhibit cancellation between the quadratic covariation and the apparent dissipation from the correction term. This is a key hyperbolic mechanism at the continuous level and ensures that the schemes are mean-square L^2-stable for a large family of numerical fluxes. Furthermore, whenever the noise coefficients σ_ℓ are divergence-free, the schemes are shown to preserve pathwise stability if the numerical fluxes are chosen carefully. Numerical experiments corroborate the stability and high-order accuracy of the suggested framework.

P4-2 Probabilistic Models

Uncertainty quantification of conservation laws with several random parameters

Dubois, Juliette; Herty, Michael; Müller, Siegfried

Technical University Berlin, Germany

Tuesday, May 26, 16:00-16:25, M17.02

This work addresses uncertainty quantification (UQ) for conservation laws with random initial conditions. We use the stochastic finite volume method (SFV), where the UQ problem is reformulated as a deterministic problem with additional variables [1]. To keep the computational cost reasonable, in [2] the SFV was combined with low-rank tensors. We introduce in [3] a new format combining full tensors for the physical space and tensor trains for the stochastic space. This hybrid format can easily be applied to any type of finite volume schemes.

In this talk, I will present numerical experiments with one dimension in physical space and several dimensions in stochastic space. The Burgers' equation and the Sod problem with random initial conditions are solved using a MUSCL scheme. We compare the format proposed in [2] and the new hybrid format. We then solve the Shu-Osher problem, and show that the hybrid format is able to capture complex features of hyperbolic problems such as shocks and waves interactions.

This is a joint work with Michael Herty (RWTH Aachen University) and Siegfried Müller (RWTH Aachen University).

[1] R. Abgrall and S. Tokareva. The Stochastic Finite Volume Method. In: Uncertainty Quantification for Hyperbolic and Kinetic Equations Springer International Publishing, 2017.

[2] S. Walton, S. Tokareva and G. Manzini. The tensor-train stochastic finite volume method for uncertainty quantification. *Journal of Computational Physics*, 538:114192, 2025.

[3] J. Dubois and M. Herty and S. Müller. High-dimensional stochastic finite volumes using the tensor train format, arXiv:2502.04868, 2026.

Tensor train discontinuous Galerkin with multiresolution-based grid adaptation for high-dimensional random hyperbolic conservation laws

Kolb, Adrian; Dubois, Juliette; Herty, Michael; Müller, Siegfried; Werthmann, Tim A.

RWTH Aachen University, Germany

Tuesday, May 26, 16:25-16:50, M17.02

Uncertainty quantification for hyperbolic conservation laws poses significant challenges, particularly due to the formation of discontinuities in both the physical and stochastic domains. Efficient and accurate numerical methods are therefore essential for computing stochastic moments of the solution. While several approaches have been proposed for random hyperbolic PDEs, many become infeasible for high-dimensional stochastic spaces due to the curse of dimensionality. One promising strategy to mitigate this is to represent the stochastic dependence using low-rank tensor decompositions. In this framework, either the full solution is represented by a single tensor train including the spatial dimensions [1], or each spatial cell carries its own tensor train compressing the stochastic dependence [2].

In this talk, we present a Discontinuous Galerkin (DG) method for random hyperbolic conservation laws, where each DG coefficient is represented as a tensor capturing the dependence on the stochastic variables. To reduce computational cost, we employ the tensor train format [3],

whose storage and arithmetic operations scale linearly in the stochastic dimension, breaking the curse of dimensionality. The time evolution is then performed directly on the tensor trains by replacing scalar operations in the DG scheme, such as flux evaluations and quadrature, with their tensor train counterparts. Moreover, rank-adaptive rounding allows direct control over the trade-off between accuracy and computational cost. Additionally, we apply multiresolution-based grid adaptation in the spatial domain driven by the stochastic moments [4]. Since each DG coefficient of a spatial cell carries a tensor train, reducing the number of cells through adaptation yields considerable computational savings. By performing the adaptation on the stochastic moments, significant details of the moments are well resolved.

Numerical experiments demonstrate the efficiency and accuracy of the proposed method.

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[1] S. Walton, S. Tokareva and G. Manzini. The tensor-train stochastic finite volume method for uncertainty quantification. *Journal of Computational Physics*, 538:114192, 2025.

[2] J. Dubois, M. Herty and S. Müller. High-dimensional stochastic finite volumes using the tensor train format, arXiv:2502.04868, 2026.

[3] I. V. Oseledets. Tensor-train decomposition. *SIAM Journal on Scientific Computing*, 33(5):2295–2317, 2011.

[4] M. Herty, A. Kolb and S. Müller. A novel multilevel approach for the efficient computation of random hyperbolic conservation laws. In *Multiscale, Nonlinear and Adaptive Approximation II*, pages 327–346. Springer Nature Switzerland, 2024.

A multilevel strategy for high-dimensional random hyperbolic conservation laws

Hinzmann, Kai; Herty, Michael; Kolb, Adrian; Müller, Siegfried

RWTH Aachen University, Germany

Tuesday, May 26, 16:50-17:15, M17.02

In this talk, we discuss high-dimensional random hyperbolic conservation laws and present an adaptive discontinuous Galerkin (DG) strategy for their efficient numerical approximation. Our concept is based on the multilevel strategy for hyperbolic conservation laws with uncertain initial data introduced in [1, 3]. In this approach, the stochastic parameter is treated as an additional spatial variable, leading to a deterministic reformulation. Rather than solving the resulting higher dimensional deterministic problem on the entire space–stochastic grid, which would be computationally prohibitive for many stochastic variables, the authors propose to separate the spatial and stochastic scales and investigate both scales individually. A set of realizations in spatial direction and stochastic slices in stochastic direction is obtained, on which a goal-oriented multiresolution-based grid adaptation strategy [2] is performed, i.e. an adaptation strategy with respect to the stochastic moments on the realizations and a weighted adaptation with respect to the probability distribution on the stochastic slices. The resulting adaptive DG solver preserves accuracy in the stochastic moments with significantly fewer cells.

However, the multiresolution analysis is limited to a three-dimensional stochastic space at most, and the rapidly increasing computational cost makes a straightforward extension to a larger number of stochastic variables infeasible. To overcome this limitation, we split the stochastic

space into single directions. This allows us to handle an arbitrary high-dimensional random hyperbolic conservation law while keeping the computational effort as low as possible. Numerical results are presented to validate our concept in the context of oceanographic applications, where we aim to predict ocean dynamics under an uncertain bottom topography. For this purpose, we investigate the stochastic moments of the shallow water equations with a bottom topography modeled by several random variables.

Acknowledgements. This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 333849990/GRK2379 (IRTG: Hierarchical and Hybrid Approaches in Modern Inverse Problems).

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[3] A. Kolb. Multiresolution-based grid adaptation for hyperbolic conservation laws with uncertain initial data. *PhD dissertation, RWTH Aachen*, DOI 10.18154/RWTH-2024-00677, 2023.

Uncertainty quantification for nonlocal traffic flow using stochastic Galerkin approximations

Ziegler, Fabian, Göttlich, Simone

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Tuesday, May 26, 17:15-17:40, M17.02

Nonlocal Traffic Flow models have been introduced to model macroscopically the effect of downstream information on the behaviour of drivers. The term 'nonlocal' refers to the dependence of the flux function on a convolution term between a kernel function and the conserved quantity. However, in the real world one needs to account for additional uncertainties in the model such as measurement errors or uncertainty in the drivers' behaviour. Thus, the distribution of vehicles depends additionally on some random input.

The idea to represent such stochastic processes by orthogonal polynomials has been employed in uncertainty quantification and inverse problems for other problems. This approach is known as stochastic Galerkin formulation with a generalized polynomial chaos (gPC) expansion. By substituting the gPC expansions of the stochastic input into the governing equations and projecting them onto the space spanned by the basis polynomials, one obtains a deterministic system of equations for the gPC coefficients.

An important benefit of polynomial chaos methods is that they exhibit spectral convergence, leading to high accuracy with relatively few expansion terms. However, these approaches can fail in the case of flows involving discontinuities with respect to the random variables. Instead one can use stochastic collocation methods which are non-intrusive and use a deterministic approximation for the Galerkin modes just as in Monte–Carlo based methods. Another problem is posed by the fact that the deterministic Jacobian of the projected system differs from the random Jacobian of the original system and hence hyperbolicity is not guaranteed.

It can be shown that for flux functions, linear with respect to the convolution, and a suitable polynomial basis, the hyperbolicity for the nonlocal traffic flow model is preserved. Furthermore,

spectral convergence is observed and the numerical results agree with the results obtained by stochastic collocation and Monte-Carlo sampling.

Stochastic nonlocal traffic flow modeling

Böhme, Timo; Göttlich, Simone; Neuenkirch, Andreas
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Tuesday, May 26, 17:40-18:05, M17.02

As an extension of the LWR model, nonlocal traffic models account for downstream information, allowing drivers to anticipate traffic conditions ahead. Such nonlocal formulations are particularly relevant for (semi-)autonomous connected vehicles, where traffic information over long distances enables an anticipatory velocity adaptation before reaching traffic congestion.

However, traffic flow is inherently subject to stochastic influences. These may stem from true randomness, such as drivers' imperfect perception of downstream velocities or measurement noise in automated vehicles. Alternatively, they may arise from the modeler's incomplete knowledge of the true velocity function including hidden variables within traffic dynamics. In [1], we proposed a stochastic nonlocal velocity-based model that introduces a class of nonlocal conservation laws with random fluxes and incorporates such uncertainty directly into the velocity function itself. This introduction of uncertainty reopens several fundamental questions. Classical results for nonlocal models (e.g., existence, uniqueness and stability of solutions) must be reassessed in the presence of random, time-dependent fluxes. Simultaneously, new challenges arise concerning the existence and characterization of the expected density, the behavior of stochastic traffic densities and the design of suitable error processes that capture these effects from a practical and realistic standpoint. Our main contribution as of [1] revolves around the stochastic nonlocal model

$$\partial_t \rho + \partial_x \big(\rho \, (W_\eta * v_\epsilon(\rho, t))\big) = 0, \tag{sNV}$$

where $\rho(t, x)$ denotes the traffic density with initial condition $\rho_0(x)$ and W_η represents the nonlocal forward-looking kernel. The distinctive feature of this formulation is that stochasticity enters through a time-dependent velocity function:

$$v_\epsilon(\rho, t) = \max\{0,\, v(\rho + \epsilon(t; \omega))\} \qquad \omega \in \Omega,$$

with a bounded, time-dependent noise process: $\epsilon(t; \omega) = \sum_k \epsilon^k(\omega) \chi_{[t^k, t^{k+1})}$. The (possibly correlated) stochastic increments $\epsilon^k(\omega)$ can be chosen flexibly within a bounded interval.

In our talk, we will first re-assess the necessary modifications for establishing pathwise well-posedness of entropy solutions to (sNV) in the presence of time-dependent stochasticity, which was formulated in Thm. 5.8 of [1]. With the classical properties of (sNV) in place, we then move to our current research, which focuses on stochastic aspects such as the existence and description of the expected density. This includes a general discussion of admissible error processes for stochastic flux systems and is accompanied by Godunov-type numerical investigations, presenting both the white-noise perturbations used in [1] as well as a generalization to auto-correlated Markovian noise, which we sample exemplarily through Jacobian-type SDE processes. Finally, we highlight the consequences of these different perturbations for flow dynamics and introduce a measure approximation designed to capture a meaningful description of the expected density, which we compare to the classical Monte Carlo average.

[1] T. Böhme, S. Göttlich and A. Neuenkirch. A nonlocal traffic flow model with stochastic velocity. *ESAIM: M2AN*, 59(1):487–518, 2025.

P5-1 Singular Limits and Large Time Behaviour

The zero capillarity limit for the Euler-Korteweg system with no-flux boundary conditions

Cacchiò, Yuri; Antonelli, Paolo

University of Vienna, Austria

Monday, May 25, 14:00-14:25, M17.23 – 2.011

In this article [1], we study the small dispersion limit of the Euler-Korteweg system in a bounded domain with no-flux boundary conditions. We exploit a relative entropy approach to study the convergence of finite energy weak solutions towards strong solutions to the compressible Euler system. Since we consider non-trivial boundary conditions, our approach needs a correction for the limiting particle density, due to the appearance of a (weak) boundary layer. We believe this approach can be adapted to study similar singular limits involving non-trivial boundary conditions.

[1] P. Antonelli and Y. Cacchiò. *The zero capillarity limit for the Euler–Korteweg system with no-flux boundary conditions.* arXiv:2510.27682, 2025.

Large bulk viscosity limit for compressible MHD equations in critical Besov spaces

Pellecchia, Giada; Ciampa, Gennaro; Donatelli, Donatella

Università degli Studi dell'Aquila, Italy

Monday, May 25, 14:25-14:50, M17.23 – 2.011

We study the large bulk viscosity limit for the compressible magnetohydrodynamics (MHD) equations in two and three space dimensions. For arbitrarily large initial data in critical Besov spaces, we prove the global well-posedness of strong solutions and establish their convergence, with explicit quantitative rates, to solutions of the incompressible MHD system as the bulk viscosity parameter tends to infinity. Our analysis shows that, for sufficiently large bulk viscosity, the potential (compressible) part of the velocity is strongly dissipated, so that the flow becomes asymptotically incompressible. As an application of this singular-limit analysis, we construct global smooth solutions to the compressible MHD equations whose magnetic field undergoes magnetic reconnection, namely a change in the topology of magnetic field lines driven by the dynamics of the flow.

[1] G. Ciampa and D. Donatelli and G. Pellecchia. Large bulk viscosity limit for compressible MHD equations in critical Besov spaces, arXiv:2602.05878, 2026.

Vanishing viscosity limit for strictly hyperbolic system of conservation laws in 1-d with nonlinear viscosity

Haspot, Boris; Janan, Animesh

Université Paris Dauphine, France

Monday, May 25, 14:50-15:15, M17.23 – 2.011

We consider the following parabolic approximation for hyperbolic system of conservation laws in 1-D with non-singular viscosity matrix $B(u)$ and $A(u)$ strictly hyperbolic,

$$u_t^\varepsilon + A(u^\varepsilon)u_x^\varepsilon = \varepsilon(B(u^\varepsilon)u_x^\varepsilon)_x.$$

We prove global in time uniform BV bound for solution u^ε to this parabolic system when $\varepsilon > 0$ provided that the initial data is small in BV and the matrix $A(u)$ and $B(u)$ commutate. We will discuss in particular different situations when the convective matrix $A(u)$ is of Temple type, triangular and the general case. We will also show that the sequence $(u^\varepsilon)_{\varepsilon>0}$ converges strongly in L^1_{loc} to a solution u of the limit strictly hyperbolic system in the conservative case. This is a joint work with Animesh Jana.

Incompressible limits at large Mach number for a reduced compressible MHD system

Wróblewska-Kamińska, Aneta

Institute of Mathematics of the Polish Academy of Sciences, Poland

Monday, May 25, 15:15-15:40, M17.23 – 2.011

We will present a singular limit problem for a reduced model for compressible non-resistive MHD. This system can also be related to a certain class of two-fluid models. By a suitable rescaling of the magnetic pressure in terms of some parameter $\varepsilon > 0$, by letting $\varepsilon \to 0$ we perform the incompressible limit while keeping the Mach number of order $O(1)$. The study is conducted in the framework of global in time finite energy weak solutions and for ill-prepared initial data. We also consider a similar problem in presence of a strong Coriolis term. The key ingredient of the proof, based on a compensated compactness argument, is the use of the transport equation (well-known in the context of two-fluid models) underlying the dynamics. Thanks to it, and differently from previous studies about the incompressible limit, we are able to identify the asymptotic of the terms of order $O(\varepsilon)$ and to characterise their dynamics; such an information is in fact crucial to obtain a closed system in the limit. This is recent joint result with Francesco Fanelli and Young-Sam Kwon.

P5-2 Singular Limits and Large Time Behaviour

Asymptotic approximations of solutions to conservation laws near singularities.

Bressan, Alberto

Penn State University, USA

Monday, May 25, 16:00-16:25, M17.23 – 2.011

Solutions to hyperbolic conservation laws can be approximated in many different ways: by vanishing viscosity, relaxations, discrete or semi-discrete numerical schemes, approximation with a nonlocal flux, etc... For some of these methods, general $\mathbf{L}^1$ convergence results are available. The talk will outline a general approach to understand the asymptotic behavior of these approximations, at points where the hyperbolic solution has a singularity. Specifically: points along a shock, or where two shocks interact, or points where a new shock forms. This leads to a wide array of open problems. Some results will be given, in the particular case of vanishing viscosity approximations.

On the vanishing viscosity limit for inhomogeneous incompressible Navier-Stokes equations on bounded domains

Schröder, Jens; Wiedemann Emil

FAU Erlangen-Nürnberg, Germany

Monday, May 25, 16:25-16:50, M17.23 – 2.011

We study the vanishing viscosity limit for the inhomogeneous incompressible Navier-Stokes equations on bounded domains with no-slip boundary condition in two or three space dimensions. We show that, under suitable assumptions on the density, we can establish the convergence in energy space of Leray-Hopf type solutions of the Navier-Stokes equation to a smooth solution of the Euler equations if and only if the energy dissipation vanishes on a boundary layer with thickness proportional to the viscosity. This extends Kato's criterion for homogeneous Navier-Stokes equations to the inhomogeneous case. We use a new relative energy functional in our proof.

Gradient regularization in viscoelasticity: analysis and asymptotic limits

AlNajjar, Aseel; Spirito, Stefano; Tzavaras, Athanasios

King Abdullah University of Science and Technology(KAUST), Saudi Arabia

Monday, May 25, 16:50-17:15, M17.23 – 2.011

We study a Kelvin-Voigt viscoelastic model incorporating higher order strain-gradient effects and nonconvex energies in several space dimensions. We derive a priori estimates that capture the dissipative structure of the problem and establish the global existence of weak solutions. We then turn to asymptotic questions, examining both the vanishing-viscosity and zero-dispersion limits; including a rate of convergence for the vanishing viscosity limit in two dimensions. We also address nonlinear dispersive effects, proving existence in the one-dimensional case. Finally, we conclude with a discussion of the additional challenges posed by physical boundaries.

Quasi-neutral limit and the mixed layer problem of Planck-Nernst-Poisson-Navier-Stokes equations for electro-hydrodynamics

Wang, Shu ; Janan, Animesh

School of Mathematics, Statistic and Mechanics,
Beijing University of Technology, China

Monday, May 25, 17:15-17:40, M17.23 – 2.011

In this talk we discuss quasi-neutral limit and the small Debye length structure stability problem of PNP-NS equations for electro-hydrodynamics. Quasi-neutrality is one basic assumption in physics such as semiconductors and plasma, which is firstly proposed by W.Van Roosbroech in Bell System Tech. J., 1950. We prove the small debye length quasi-neutral limit of the initial-boundary value problem for PNP-NS system for physicochemical hydrodynamics with the differential diffusion coefficients for two kinds of charges and the general ill-prepared smooth initial data, and obtain the convergence rate of the approximating solution having the form of the boundary layer, the initial layer and the mixed layer to the the exact solution of the system when the Debye length tends to zero, where the mixed layer functions have only the algebraic decay rates with respect to the fast variables. Thus, the rigorous quasineutrality theory of the electric-diffusion of ions in physicochemical hydrodynamics is established.

P5-3 Singular Limits and Large Time Behaviour

Dissipative structure of higher order regularizations of hyperbolic systems of conservation laws in several space dimensions

Plaza, Ramón G.; Angeles, Felipe; Valdovinos, José M.

Universidad Nacional Autónoma de México, Mexico

Tuesday, May 26, 14:00-14:25, M17.23 – 2.011

In this talk I will present recent results on the dissipative structure of regularizations of any order of hyperbolic systems of conservation laws in several space dimensions. It is proved that the seminal equivalence theorem by Kawashima and Shizuta (Hokkaido Math. J. 14, 1985, no. 2, 249-275), which relates the strictly dissipative structure of second-order (viscous) systems to a genuine coupling condition of algebraic type, can be extended to higher-order multidimensional systems. For that purpose, the symbolic formulation of the genuine coupling condition by Humpherys (J. Hyperbolic Differ. Equ. 2, 2005, no. 4, 963-974) for linear operators of any order in one dimension, is adopted and extrapolated. Therefore, the concepts of symbol symmetrizability and genuine coupling are extended to the most general setting of differential operators of any order in several space dimensions. Applications to many viscous-dispersive systems of physical origin, such as compressible viscous-capillar fluids of Korteweg type, the dispersive Navier-Stokes-Fourier system and the equations of quantum hydrodynamics, illustrate the relevance of this extension. This a joint work with Felipe Angeles (Universidad Nacional Autónoma de México, Mexico) and José M. Valdovinos (Université de Toulouse, France).

Formal derivation of a stratified compressible two-phase flow model

Le Vourc'h, Pierrick

Université de Montpellier, Inria ANGUS team, France

Tuesday, May 26, 14:25-14:50, M17.23 – 2.011

From the combustion chamber of space launchers to circuits of pressurized water reactors of nuclear plants, the understanding of compressible multiphase flows has become a real challenge and a crucial domain of research during the last decades. Because of their complexity and the limited capacity of numerical computations, compressible multiphase flows are often described by macroscopic models obtained by averaging the Navier-Stokes equations. Furthermore, the source terms governing the relaxation to thermodynamical equilibrium between the phases (pressure, velocity, temperature) are closed by empirical relations, see [2, 4] for instance. It is a key issue to provide a mathematical theory of derivation of inviscid compressible multiphase models with no empirical closures. Starting from multiphase Navier-Stokes models, mathematical methodologies may enable to derive original average multiphase models, or to validate existing models.

In this talk, I will present a formal derivation of an averaged model for a stratified compressible two-phase flow in a thin domain [5]. The fluid layers being thin, we use a dimension reduction technique to obtain our final model [3, 6]. More precisely, we average the two sets of compressible Navier-Stokes equations — one for each fluid — in the stratification direction. While standard procedures seem to be profoundly related to one-velocity models, our approach suppresses this constraint. However, it is important to note that this modelling process necessarily leads to one-pressure models, as in [2, 4]. We are also able to recover one-velocity, one-pressure models when we study the standard framework — with continuity of the velocity field at the interface. Our derivation process also adapts to the energy equation in the Navier-Stokes-Fourier case, enabling the obtention of either one or two-temperature models.

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[2] Donald A. Drew and Stephen L. Passman. *Theory of Multicomponent Fluids*, volume 135 of Applied Mathematical Sciences. Springer, New York, NY, 1999.

[3] Jean-Frédéric Gerbeau and Benoît Perthame. Derivation of viscous Saint-Venant system for laminar shallow water; numerical validation. *Discrete & Continuous Dynamical Systems – B*, 1(1):89–102, 2001.

[4] Mamoru Ishii and Takashi Hibiki. *Thermo-Fluid Dynamics of Two-Phase Flow*. Springer, New York, 2nd edition, 2011.

[5] Pierrick Le Vourc'h, Khaled Saleh, and Nicolas Seguin. Averaged models for compressible two-phase stratified flows on thin domains. June 2025. *arXiv*:2506.08542.

[6] H. Bruce Stewart and Burton Wendroff. Two-phase flow: Models and methods. *Journal of Computational Physics*, 56(3):363–409, December 1984.

Hyperbolic regularization of free-surface flow models

Koellermeier, Julian

Ghent University, Belgium

University of Groningen, the Netherlands

Tuesday, May 26, 14:50-15:15, M17.23 – 2.011

Shallow Water Moment Equations (SWME) are reduced-order models for free-surface flows, employing a vertical velocity expansion and deriving additional moment equations for the expansion coefficients. This adds accuracy in comparison to the simpler but less accurate Shallow Water Equations (SWE), which only incorporate depth-average (mean) velocity information. However, different versions of the SWME typically fail to simultaneously satisfy key analytical properties: hyperbolicity, accuracy, correct momentum balance, and interpretable steady states.

In this presentation, we first review the existing types of SWME and highlight their properties. Then, we address their limitations by discussing a novel hyperbolic regularization approach in primitive variables. We shows that by transforming the system to primitive variables and subsequent regularization, we derive new hierarchies of hyperbolic SWME models: the Primitive regularization for Hyperbolic Shallow Water Moment Equations (PHSWME) and the Primitive Moment regularization for Hyperbolic Shallow Water Moment Equations (PMHSWME). These new models preserve the momentum equation, ensure global hyperbolicity, and admit analytical steady states, while at the same time maintaining high accuracy compared to the standard (non-hyperbolic) SWME. We analytically prove hyperbolicity for both new models and derive their analytical steady states. Numerical simulations of a dam-break test show the accuracy of the new models.

The Cattaneo-Christov system for compressible fluid flow in several space dimensions: Well-posedness and non-hyperbolicity

Angeles, Felipe

Department of Mathematics and Mechanics, Institute of Applied Mathematics and Systems UNAM, Mexico

Tuesday, May 26, 15:15-15:50, M17.23 – 2.011

We consider the coupling between the equations of motion for a compressible fluid in several space dimensions with the objective formulation of the Maxwell-Cattaneo law proposed by Christov,

$$\tau\left[q_t + v\cdot\nabla q - q\cdot\nabla v + (\nabla\cdot v)q\right] + q = -\kappa\nabla\theta,$$

where q is the heat flux, θ the temperature, v the velocity field, κ the thermal conductivity and τ the relaxation time. Christov's equation is a frame indifferent formulation of the classical model of Cattaneo that allows for the heat flux to be eliminated to obtain a single hyperbolic equation for the temperature. The resulting system is written in quasilinear form for the state variables: density, velocity, temperature and heat flux.

We show that the inviscid version of this system is not hyperbolic due to the presence of the Lie–Oldroyd upper-convected material derivative involved in Christov's formulation. As consequence, the system of equations does not admit a Friedrichs symmetrizer. Nonetheless, we show that the initial value problem for the viscous *Cattaneo-Christov system* for compressible fluid flow is locally well posed in H^s with $s > \frac{d}{2} + 1$.

P5-4 Singular Limits and Large Time Behaviour

L^2-contraction for viscous-dispersive shock waves of KdV–Burgers equation

Eun, Namhyun

Korea Advanced Institute of Science and Technology (KAIST), Republic of Korea

Tuesday, May 26, 16:00-16:25, M17.23 – 2.011

It is well known that the Korteweg–de Vries–Burgers (KdVB) equation admits traveling wave solutions, referred to as viscous-dispersive shock waves. When the dispersion dominates viscosity, the associated shock profiles exhibit infinitely many oscillations. Although the KdVB equation serves as a canonical model incorporating nonlinearity, viscosity, and dispersion, the stability of these shock profiles—especially in the oscillatory regime—remains poorly understood.

In this talk, we discuss detailed structural properties of the shock profiles, and then prove an L^2-contraction property under arbitrarily large perturbations, up to a time-dependent shift. Finally, we justify the zero viscosity-dispersion limit.

This is a collaborative work with Geng Chen (University of Kansas), Moon-Jin Kang (KAIST), and Yannan Shen (University of Kansas).

On the asymptotic behavior of irrotational steady Euler equations

Wang, Tian-Yi

School of Mathematics and Statistics, Wuhan University of Technology, China

Tuesday, May 26, 16:25-16:50, M17.23 – 2.011

This talk concerns the asymptotic behavior of steady irrotational Euler flows. We begin by establishing the far-field convergence rate of order $|x|^{-(n-1)}$ through a refined maximum principle argument, which is optimal for the largely open nozzle problem.. For the airfoil problem, the slip boundary condition induces a first-order cancellation, yielding an improved velocity convergence rate of $|x|^{-n}$ for $n \geq 3$. The asymptotic expansion formula for subsonic flows at infinity is derived via the Kelvin transformation and Schauder estimates near the origin, with special attention to the two-dimensional case. An explicit example is constructed to confirm the optimality of these rates. Finally, we present recent progress on the two-dimensional general largely open nozzle problem.

Long time behavior of stochastic isentropic Euler equation with linear damping

Dai, Rongyi

University of California, Berkeley, United States of America

Tuesday, May 26, 16:50-17:15, M17.23 – 2.011

We study the one dimensional isentropic Euler equations with linear (frictional) damping and adiabatic constant $\gamma > 1$, subject to a multiplicative, white-in-time stochastic forcing. In the deterministic case, there are rigorous mathematical justifications of porous medium equation and Darcy's law as the long time asymptotic limit of the isentropic Euler equation with linear damping in various settings. We provide an extension of this result to the stochastic case in the setting of an initial-boundary-value problem.

We first establish the global-in-time existence of L^∞ martingale solutions that satisfy an appropriate entropy inequality. We then prove that, under suitable assumptions on the noise, these

solutions converge almost surely to a deterministic constant stationary state with an exponential rate in time. The constant stationary state coincides with that of an initial-boundary-value problem of the deterministic porous medium equation.

Our approach relies on adapting the entropy dissipation method to the stochastic setting on the viscous approximation level, deriving higher moment estimates for the approximate solutions and their entropy, and applying recent results on pathwise convergence of stochastic integrals. To the best of our knowledge, this work provides the first rigorous pathwise convergence result for the long-time behavior of solutions to the stochastic isentropic compressible Euler equations with linear damping.

Measure solutions, smoothing effect, and deterministic particle approximation for a conservation law with non-local flux

Fagioli, Simone

University of L'Aquila, Italy

Tuesday, May 26, 17:15-17:40, M17.23 – 2.011

We consider a class of non-local conservation laws with an interaction kernel supported on the negative real half-line and featuring a decreasing jump at the origin. We provide, for the first time, an existence and uniqueness theory for said model with initial data in the space of probability measures. Our concept of solution allows us to sort a lack of uniqueness problem which we exhibit in a specific example. Our approach uses the so-called quantile, or pseudo-inverse, formulation of the PDE, which has been largely used for similar types of non-local transport equations in one space dimension. Partly related to said approach, we then provide a deterministic particle approximation theorem for the equation under consideration, which works for general initial data in the space of probability measures with compact support. As a crucial step in both results, we use that our concept of solution (which we call dissipative measure solution) implies an instantaneous measure-to-L^∞ smoothing effect, a property which is known also to be featured by local conservation laws with genuinely non-linear fluxes.

Joint work with M. Di Francesco and E. Radici (University of L'Aquila)

P6-1 Structure Preserving Methods

On the heat GLM system and its compatible discretization

del Río Martín, Laura; Dhaouadi, Firas; Dumbser, Michael

INRIA center of the University of Bordeaux, France

Monday, May 25, 14:00-14:25, M17.25 – 2.017

The Generalized-Lagrangian Multiplier (GLM) cleaning, introduced by Munz et al. in 2000, is a practical strategy for enforcing div- and curl-free constraints. In this talk, I will present a rigorous formulation of GLM curl-cleaning for a prototype system of heat propagation. We show that the equations can be rigorously derived from an underlying variational principle and that the resulting system is symmetric hyperbolic. The system preserves the initial curl of its vector fields at the continuous level and is asymptotically consistent with Fourier's law of heat conduction. In order to verify these properties numerically, a new semi-implicit compatible numerical scheme on staggered grids has been developed, which provably preserves the asymptotic Fourier limit, exactly preserves the Lyapunov-function decay, and is curl preserving. These results are confirmed in practice in a set of carefully designed test cases.

Construction of asymptotic preserving schemes for kinetic models using upwind summation-by-parts operators

Elghawi, Salim

University of Kassel, Germany

Monday, May 25, 14:25-14:50, M17.25 – 2.017

When studying certain phenomena that involve the interaction of a large number of particles, one encounters the concept of kinetic models. In diffusion scaling, these models have the form

$$\varepsilon \partial_t f + v \nabla_x f = \frac{1}{\varepsilon} \mathcal{Q}(f, f)\,.$$

The most famous kinetic model which arises during the study of rarefied gases is the Boltzmann equation. The parameter ε then represents the Knudsen number which is the ratio of the mean free path with respect to the characteristic physical length scale. When $\varepsilon \to 0$, the kinetic model converges towards a macroscopic limit equation, e.g. the compressible Euler equations. Explicit time discretization schemes for the kinetic model become restrictive and computationally expensive, as the explicit time step size Δt scales as $O(\varepsilon)$. Implicit schemes on the other hand, are also computationally demanding as we are often required to solve large systems of equations. Therefore, we consider implicit-explicit (IMEX) schemes. Furthermore, we need to construct schemes such that for $\varepsilon \to 0$, and fixed discretization parameters Δt and Δx, the resulting discretization is consistent with the macroscopic equation given by the asymptotics. Such schemes are said to be asymptotic preserving (AP). A useful tool in discretizing derivatives in space is given by the concept of upwind summation-by-parts (SBP) operators combined with simultaneous approximation terms (SATs) for the imposition of boundary conditions. The SBP-SAT framework allows to transfer continuous energy stability results to the discrete setting. In this talk, we will consider energy stable SAT boundary treatment for simplified linear kinetic models combined with central and upwind SBP schemes in space. Continuous, semidiscrete and fully discrete energy stability in the linear case will be shown for this approach.

A second-order asymptotic-preserving IMEX scheme for the isentropic compressible Cahn-Hilliard-Navier-Stokes equations

Martorell, Andreu; Mulet, Pep; Yáñez, Dionisio F.

University of Valencia, Spain

Monday, May 25, 14:50-15:15, M17.25 – 2.017

We introduce a second-order asymptotic-preserving implicit-explicit (IMEX) finite difference scheme on staggered grids for the isentropic compressible Cahn-Hilliard-Navier-Stokes equations.

This model involves a challenging interplay between convective and diffusive terms, with diffusion governed by second- to fourth-order spatial differential operators. Moreover, in low Mach number regimes, fast acoustic waves impose severe stability constraints on standard IMEX schemes [3], while the system asymptotically approaches towards its incompressible counterpart.

To address these difficulties, we adopt a pressure-splitting strategy that separates stiff and non-stiff contributions [1]. By treating the stiff pressure and high-order diffusive terms implicitly, the resulting time-step restriction is determined solely by the convective subsystem and remains independent of the Mach number. The proposed scheme ensures a stable coupling between density and velocity [2], yielding a robust symmetric discretization that consistently captures the incompressible limit.

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A new asymptotic-preserving dual formulation finite-volume method for the compressible Euler equations

Chertock, Alina

North Carolina State University, USA

Monday, May 25, 15:15-15:40, M17.25 – 2.017

It is well known that low-Mach-number flows pose several significant challenges for numerical simulations due to their multiscale structure: slow material waves coexist with fast acoustic waves whose speeds increase as the Mach number decreases. As a consequence, explicit numerical schemes face severe time-step restrictions tied to the largest wave speeds, leading to inefficiency and excessive numerical diffusion. At the same time, fully implicit approaches may oversmear material waves and fail to resolve the correct zero-Mach limit. This motivates the development of numerical methods that remain stable and accurate across all Mach number regimes.

In this talk, we present new numerical methods for the compressible Euler equations that achieve this goal through an asymptotic-preserving dual-formulation framework. Departing from conventional conservative hyperbolic flux-splitting strategies, we reformulate the system

in primitive (nonconservative) variables and employ a nonconservative hyperbolic splitting. The resulting system is discretized using a semi- implicit scheme: stiff components are advanced using second-order implicit central differences, while nonstiff components are treated with an explicit, second-order path-conservative central-upwind method. The asymptotic-preserving property is enforced by computing the pressure at each time level via a Poisson-type elliptic equation. In parallel, the conservative formulation is evolved with a semi-discrete central- upwind scheme, and a tailored post-processing step selects the appropriate numerical solution depending on the Mach-number regime, ensuring a consistent low-Mach limit while preserving sharp, physically relevant features at higher Mach numbers. Numerical experiments confirm second-order accuracy and demonstrate that the stability constraint is independent of the Mach number, highlighting the proposed approach as a robust and efficient in all Mach number regimes.

P6-2 Structure Preserving Methods

Asymptotic-preserving discontinuous Galerkin method for the Navier Stokes equations with low Mach number

Zhang, Jiahui

Johannes Gutenberg University of Mainz, Germany

Monday, May 25, 16:00-16:25, M17.25 – 2.017

In this talk, a high-order asymptotic-preserving scheme is developed for the isentropic Navier-Stokes equations with low Mach number. The high-order accuracy is achieved by combining the discontinuous Galerkin method for spatial discretization with an implicit-explicit (IMEX) strategy in time. Our approach involves splitting the flux into an implicitly treated linear hyperbolic stiff acoustic part and an explicitly treated nonlinear non-stiff convective part. The rigorous theoretical analysis demonstrates the asymptotic-preserving property of the proposed scheme, while a series of numerical experiments are conducted to validate its desirable properties and confirm its high-order accuracy. If time permits, we will also present the results for continuous data assimilation. This research was funded by Gutenberg Research Fellowship and TRR146 Multiscale Simulation Methods for Soft Matter Systems.

An asymptotic-preserving scheme for the low-Mach-number Euler equations via dissipative measure-valued solutions

Koottungal Revi, Arun; Krishnamurthy, Amogh; Lukáčová-Medviďová, Mária

School of Mathematics, Indian Institute of Science Education and Research Thiruvananthapuram, India

Monday, May 25, 16:25-16:50, M17.25 – 2.017

We propose and analyze a new asymptotic-preserving (AP) finite-volume scheme for the multidimensional compressible barotropic Euler equations in the low-Mach-number regime. The scheme is based on a stabilized upwind numerical flux, with stabilization proportional to the stiff pressure gradient, and is shown to be conditionally energy stable and consistent.

The asymptotic-preserving property is established rigorously within the framework of dissipative measure-valued (DMV) solutions, which provides a unified setting for the simultaneous limits of vanishing mesh size and Mach number. For fixed Mach number, we prove that the numerical solutions converge weakly, as the mesh parameter tends to zero, to a DMV solution of the compressible Euler equations. As the Mach number vanishes, these DMV solutions converge to a classical solution of the incompressible Euler system.

Conversely, for fixed mesh size, the scheme yields an energy-stable and consistent discretization of the incompressible Euler equations in the low-Mach limit. In this regime, the numerical solutions converge weakly, as the mesh parameter vanishes, to a DMV solution of the incompressible Euler system. By invoking the weak–strong uniqueness principle, we conclude that the DMV and classical solutions coincide, thereby proving the AP property of the scheme.

An extensive numerical study is presented to support the theoretical analysis. Using the relative entropy with respect to strong incompressible Euler solutions, we obtain error estimates that clearly demonstrate the asymptotic-preserving behavior of the proposed scheme.

Asymptotically compatible entropy-consistent discretization for a class of nonlocal conservation laws

Huang, Kuang; De Nitti, Nicola

The Chinese University of Hong Kong, Hong Kong SAR, China

Monday, May 25, 16:50-17:15, M17.25 – 2.017

We consider a class of nonlocal conservation laws modeling traffic flows, given by $\partial_t \rho_\varepsilon + \partial_x(V(\rho_\varepsilon * \gamma_\varepsilon)\rho_\varepsilon) = 0$ with a suitable convex kernel γ_ε, and its Godunov-type numerical discretization. We prove that, as the nonlocal parameter ε and mesh size h tend to zero simultaneously, the discrete approximation $W_{\varepsilon,h}$ of $W_\varepsilon := \rho_\varepsilon * \gamma_\varepsilon$ converges to the entropy solution of the (local) scalar conservation law $\partial_t \rho + \partial_x(V(\rho)\rho) = 0$, with an explicit convergence rate estimate of order $\varepsilon + h + \sqrt{\varepsilon\, t} + \sqrt{h\, t}$. In particular, with an exponential kernel, we establish the same convergence result for the discrete approximation $\rho_{\varepsilon,h}$ of ρ_ε, along with an L^1-contraction property for W_ε. The key ingredients in proving these results are uniform L^∞- and TV-estimates that ensure compactness of approximate solutions, and discrete entropy inequalities that ensure the entropy admissibility of the limit solution.

An all-Froude semi-implicit solver for two-dimensional shallow-water

Ramírez Fuentes, José Ignacio; Muñoz Ruiz, María de la Luz; Escalante Sánchez, Cipriano; Morales de Luna, Tomás

University of Málaga (UMA), Spain

Monday, May 25, 17:15-17:40, M17.25 – 2.017

This work presents a two-dimensional extension of the semi-implicit finite-volume relaxation scheme originally introduced for the one-dimensional shallow-water equations in [1]. The goal is to retain the main advantages of the 1D formulation—in particular, outperform in low-Froude regimes and well-balanced properties—while providing an efficient solver for 2D flows.

First, the core ideas behind the relaxation approach are revisited. The scheme consists of a splitting that separates the acoustic and transport subsystems. The acoustic step, built from the Riemann invariants of the associated relaxed system, targets the gravity waves through an implicit update, while the transport step advances the material waves explicitly. This decomposition provides a clear route to construct the numerical fluxes and leads to a two-dimensional scheme.

Finally, a consistent 2D extension is designed within a dimensional-splitting framework. The resulting solver provides an efficient 2D method that preserves the semi-implicit character and the well-balanced features of the original approach.

Invariant region preserving schemes for mixed hyperbolic-parabolic SPDEs

Maroni, Kristian; Christiansen, Thomas; Karlsen, Kenneth H.; Ulusoy, Süleyman

University of Oslo, Norway

Monday, May 25, 17:40-18:05, M17.25 – 2.017

Mixed hyperbolic-parabolic SPDEs encapsulate a large spectrum of equations that have broad applications in various fields. When the diffusion matrix is positive semi-definite, the resulting

SPDE can experience strong degeneracy, inducing non-uniqueness and shock waves. To ensure uniqueness of weak solutions, appropriate entropy conditions that allow for discontinuous solutions are required. In addition, one has to account for stochastic perturbations. It is then a nontrivial task to design numerical schemes that preserve pointwise invariant regions, where the entropy solution remains inside the same domain as the initial condition. In this talk we showcase two invariant region preserving numerical schemes that relies on an operator-splitting of the deterministic and stochastic part of the equation

$$du + \left(\sum_{i=1}^{d} \partial_{x_i} f_i(u) - \sum_{i,j=1}^{d} \partial_{x_i} \big(a_{ij}(u) \partial_{x_j} u \big) \right) dt = \sigma(u)\, dB,$$

where a is a symmetric and positive semi-definite matrix, f is a non-linear flux, and $\sigma(u)dB$ is a cylindrical Wiener noise. The first scheme discretizes the deterministic part with a local discontinuous Galerkin method, while the stochastic part is discretized using a Lamperti transformation combined with an explicit Runge–Kutta method. Meanwhile, for the second scheme, the deterministic part is discretized by a finite difference scheme with cross derivative correctors, while the stochastic part is discretized with an artificial barrier method combined with the Euler–Maruyama scheme. The splitting approach is justified by theoretical results that shows that semi-discrete splitting solutions converge to the unique entropy solution of more general stochastic convection-diffusion equations. Moreover, numerical experiments confirm that the numerical schemes are stable and preserve both positivity and invariant regions.

Acknowledgements. This research was funded by the Research Council of Norway under project 351123 (NASTRAN).

P6-3 Structure Preserving Methods

A simple and general framework for the construction of exactly div-curl-grad compatible discontinuous Galerkin finite element schemes on unstructured simplex meshes

Dumbser, Michael; Abgrall, Rémi; Maire, Pierre-Henri

University of Trento, Italy

Tuesday, May 26, 14:00-14:25, M17.25 – 2.017

We introduce a new family of discontinuous Galerkin (DG) finite element schemes for the discretization of first order systems of hyperbolic partial differential equations (PDE) on unstructured simplex meshes in two and three space dimensions that respect the two basic vector calculus identities exactly also at the discrete level, namely that the curl of the gradient is zero and that the divergence of the curl is zero. The key ingredient here is the construction of two compatible discrete nabla operators, a primary one and a dual one, both defined on general unstructured simplex meshes in multiple space dimensions. Our new schemes extend existing cell-centered finite volume methods based on corner fluxes to arbitrary high order of accuracy in space. An important feature of our new method is the fact that only two different discrete function spaces are needed to represent the numerical solution, and the choice of the appropriate function space for each variable is related to the origin and nature of the underlying PDE. The first class of variables is discretized at the aid of a discontinuous Galerkin approach, where the numerical solution is represented via piecewise polynomials of degree N and which are allowed to jump across element interfaces. This set of variables is related to those PDE which are mere consequences of the definitions, derived from some abstract scalar and vector potentials, and for which involutions like the divergence-free or the curl-free property must hold if satisfied by the initial data. The second class of variables is discretized via classical continuous Lagrange finite elements of approximation degree $M = N+1$ and is related to those PDE which can be derived as the Euler-Lagrange equations of an underlying variational principle. The primary nabla operator takes as input the data from the FEM space and returns data in the DG space, while the dual nabla operator takes as input the data from the DG space and produces output in the FEM space. The two discrete nabla operators satisfy a discrete Schwarz theorem on the symmetry of discrete second derivatives. From there, both discrete vector calculus identities follow automatically. We apply our new family of schemes to three hyperbolic systems with involutions: the system of linear acoustics, in which the velocity field must remain curl-free and the vacuum Maxwell equations, in which the divergence of the magnetic field and of the electric field must remain zero. In our approach, only the magnetic field will remain exactly divergence free. As a third model we study the Maxwell-GLM system of Munz *et al.*, which contains a unique mixture of curl-curl and div-grad operators and in which the magnetic field may be either curl-free or divergence-free, depending on the choice of the initial data. In all cases we prove that the proposed schemes are exactly total energy conservative and thus nonlinearly stable in the L^2 norm.

A thermodynamically compatible scheme for continuum mechanics with torsion

A. Lucca, I. Peshko, M. Dumbser

University of Trento, Italy

Tuesday, May 26, 14:25-14:50, M17.25 – 2.017

In the proposed talk, we will present a thermodynamically compatible finite volume scheme for solving nonlinear systems of hyperbolic partial differential equations (PDEs), as applied to continuum mechanics and complex materials. Specifically, we will consider the general Godunov–Peshkov–Romenski (GPR) model, which has been extended to account for the rotational degrees of freedom of irregular small-scale vortex dynamics [1]. This unified model describes elastic and elasto-plastic solids in the nonlinear large-strain regime, as well as viscous and inviscid heat-conducting fluids, all under a single class of first-order hyperbolic equations. This formulation uses a geometric description of mechanics and is based on the theory of symmetric hyperbolic thermodynamically compatible (SHTC) equations. The PDE system indeed satisfies an additional conservation law for total energy, obtained via a suitable linear combination of the other equations based on Godunov variables. To preserve this structure, we propose a semi-discrete finite volume method based on the general framework introduced by Abgrall [2]. Unlike in most traditional schemes, here the PDE for the entropy density is solved as a primary evolution variable. Conversely, total energy conservation is obtained as a direct consequence of thermodynamically compatible discretisation of the other equations. Consequently, the proposed HTC scheme satisfies the entropy inequality, including the entropy production term, by construction, and can be proven to be nonlinearly stable in the energy norm. We will then present some computational results obtained using the aforementioned HTC scheme in one and two spatial dimensions, considering both the fluid and solid limits of the governing PDEs.

[1] I. Peshkov, E. Romenski, and M. Dumbser. Continuum mechanics with torsion. *Continuum Mechanics and Thermodynamics*, 31:1517–1541, 2019.

[2] R. Abgrall. A general framework to construct schemes satisfying additional conservation relations. Application to entropy conservative and entropy dissipative schemes. *Journal of Computational Physics*, 372:640–666, 2018.

A Hamiltonian approach to entropy-stable and well-balanced schemes

Ersing, Patrick; Winters, Andrew R.

Linköping University, Sweden

Tuesday, May 26, 14:50-15:15, M17.25 – 2.017

Many fundamental models in fluid dynamics possess an underlying Hamiltonian structure, where the evolution is governed by energy principles. In the Hamiltonian framework, steady-state solutions can be characterized as conditional extrema of the total energy subject to the invariants of motion. This variational viewpoint provides a natural framework for analyzing the preservation and stability of steady-state solutions at both the continuous and discrete levels.

From the continuous Hamiltonian theory, we derive criteria under which numerical methods preserve steady states by respecting the conservation of total energy and specific invariants at the discrete level. In particular, for models where the total energy also serves as a convex mathematical entropy function, it is shown that discrete entropy stability combined with conservation of invariants directly implies preservation of steady-state solutions.

This methodology is then applied to construct a provably entropy-stable and well-balanced discontinuous Galerkin method for the shallow water equations with bottom topography. Numerical experiments confirm the theoretical results, demonstrating entropy stability and steady-state preservation. Finally, we discuss open questions and outline possible extensions to other fluid dynamics models.

How to make fully discrete entropy stability of three-point well-balanced finite-volume schemes

Morales de Luna, Tomás Berthon, Christophe; Castro, Manuel; Martaud, Ludovic;

Universidad de Málaga, Spain

Tuesday, May 26, 15:15-15:40, M17.25 – 2.017

We introduce a robust and general strategy to enforce well-balancing and discrete entropy stability in first-order finite volume schemes for systems of balance laws and hyperbolic systems with non-conservative products.
While well-balance can be achieved by using a general technique based on the fluctuations with respect to stationary solutions, an artificial viscosity technique is proposed to modify the scheme. The key point is the construction of a specific viscosity coefficient that simultaneously preserves the well-balanced property and guarantees entropy dissipation at the discrete level. To the best of our knowledge, this is the first time such a general and constructive procedure is presented, offering a unified framework to develop stable and physically relevant numerical methods for a wide range of balance law systems.
The effectiveness of this procedure is demonstrated through various numerical experiments, including shallow water and two-layer flow systems, highlighting its ability to provide accurate estimations, providing a practical and theoretically sound procedure and offering potential for future extensions to higher-order methods.

P6-4 Structure Preserving Methods

Well-balanced semi-implicit relaxed finite volume schemes for hyperbolic balance laws

I. Gómez-Bueno, M.J. Castro, A. Thomann

Universidad de Málaga, Spain

Tuesday, May 26, 16:00-16:25, M17.25 – 2.017

We propose a class of well-balanced semi-implicit relaxed finite volume schemes for hyperbolic systems of balance laws, aimed at multi-scale regimes in which fast waves can impose severe stability constraints on fully explicit discretizations. The construction builds upon semi-implicit relaxation ideas for multi-scale hyperbolic conservation laws in the spirit of the Jin–Xin relaxation framework, where the flux is decomposed into subsystems separating relevant scales and stiff components are treated implicitly, yielding a linear structure in the relaxation variables and, consequently, a linearly implicit update [1, 2].

To incorporate well-balancedness into this semi-implicit relaxed setting, we draw on established strategies from the literature. In particular, we consider hydrostatic-type reconstructions and path-conservative formulations based on suitable path choices to capture the relevant equilibria [3], as well as operator-based well-balanced reconstructions combined with compatible source-term discretizations [4, 5]. In this way, the resulting scheme is well-balanced for all stationary solutions of the considered balance law, or at least for a relevant family of steady states.

As a representative application, we consider shallow water models, which offer a standard setting to assess equilibrium preservation and multi-scale robustness in balance-law dynamics.

[1] S. Jin and Z. Xin, The relaxation schemes for systems of conservation laws in arbitrary space dimensions, *Communications on Pure and Applied Mathematics*, 48(3):235–276, 1995.

[2] A. Thomann, Semi-implicit relaxed finite volume schemes for hyperbolic multi-scale systems of conservation laws, *Journal of Computational Physics*, 539:114263, 2025.

[3] M. J. Castro, T. Morales de Luna, and C. Parés, Well-balanced schemes and path-conservative numerical methods, in *Handbook of Numerical Analysis*, vol. 18, pp. 131–175, Elsevier, 2017.

[4] M. J. Castro and C. Parés, Well-Balanced High-Order Finite Volume Methods for Systems of Balance Laws, *Journal of Scientific Computing*, 82:42, 2020.

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Development of discontinuous Galerkin methods for hyperbolic systems that preserve a divergence or a curl constraint

Perrier, Vincent; Jung, Jonathan

INRIA, team Cagire & LMAP, Université de Pau et des Pays de l'Adour, Pau, France

Tuesday, May 26, 16:25-16:50, M17.25 – 2.017

Some hyperbolic systems are known for ensuring implicitly a divergence or a curl constraint. For example, the wave system ensures the preservation of the vorticity, or the Maxwell system and the Magnetohydrodynamics system respectively ensure a preservation of the divergence of the magnetic field, and an initially divergence free magnetic field.

These constraints are usually complicated to preserve at the discrete level, and it is usual to resort to so-called staggered schemes, e.g. MAC scheme for ensuring the incompressibiity constraint, or the Yee scheme for electromagnetism, and their higher order variants based on Nédélec or Raviart-Thomas finite elements.

This talk will start from a remark that was done years ago in the context of low Mach number problems [1, 2], and based on these results, we will explain in which conditions, and in which sense divergence and curl constraints can be preserved with finite volume schemes on triangular meshes.

Then, based on ideas developed within the "finite element exterior calculus" [7], and on the distributional de-Rham complexes introduced in [6], I will show how to extend the low order case on triangular meshes to the high order case, and to the quadrangular meshes case. Results of this talk were published in [3, 4, 5].

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[2] Hervé Guillard. On the behavior of upwind schemes in the low Mach number limit. IV: P0 approximation on triangular and tetrahedral cells. *Computers & Fluids*, 38(10):1969–1972, 2009.

[3] Jonathan Jung and Vincent Perrier. A curl preserving finite volume scheme by space velocity enrichment. Application to the low Mach number accuracy problem. *Journal of Computational Physics*, 515:113252, 2024.

[4] Vincent Perrier. discrete de-Rham complex involving a discontinuous finite element space for velocities: the case of periodic straight triangular and Cartesian meshes. *Annales Henri Lebesgue*, 8:417–452, 2025.

[5] Vincent Perrier. Development of discontinuous Galerkin methods for hyperbolic systems that preserve a curl or a divergence constraint: the case of linear systems. *Journal of Computational Physics*, page 114445, 2025.

[6] Martin Werner Licht. Complexes of discrete distributional differential forms and their homology theory. *Foundations of Computational Mathematics*, 17(4):1085–1122, 2017.

[7] Douglas Norman Arnold. *Finite element exterior calculus.* SIAM, 2018.

Numerical study of polydisperse sedimentation in complex geometries using an invariant-region-preserving scheme

Barajas-Calonge, Juan

Universidad del Bío-Bío, Chile

Tuesday, May 26, 16:50-17:15, M17.25 – 2.017

In this talk, a two-dimensional model for the settling of polydisperse suspensions in general geometries is presented. The model consists of a system of conservation laws in two dimensions that describes the concentrations of particles, coupled with a Stokes-type equation governing the velocity of the mixture. We propose a numerical scheme in which the Stokes equations are discretized on a primal triangular mesh using a finite element approach, while the concentration equations are approximated on a dual diamond mesh using a finite volume scheme. The solution

vector is required to take values in a set of physically relevant states (i.e., its components are non-negative and sum to at most a prescribed maximum value). We show that this set, known as the invariant region, is preserved by the numerical solutions produced by the proposed scheme. To ensure this property, and motivated by [1], a pair of linear scaling limiters is applied to a second-order MUSCL reconstruction, yielding invariant-region-preserving (IRP) second-order polynomial reconstructions. Finally, we describe the coupling algorithm and present numerical simulations of scenarios of interest.

This presentation is based on joint research with R. Bürger (Universidad de Concepción, Chile), P. Mulet (Universitat de València, Spain) and L.M. Villada (Universidad del Bío-Bío, Chile).

[1] X. Zhang, C.-W. Shu. On maximum-principle-satisfying high order schemes for scalar conservation laws *Journal of Computational Physics*, 229:3091-3120, (2010).

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[3] J. Barajas-Calonge, R. Bürger, P. Mulet, and L.M. Villada. Invariant-region-preserving WENO schemes for one-dimensional multispecies kinematic flow models, *Journal of Computational Physics*, 114081 (2025).

[4] J. Barajas-Calonge, R. Bürger, P. Mulet and L.M. Villada. A second-order invariant-region-preserving scheme for a transport-flow model of polydisperse sedimentation. Preprint 2025-16, CI^{2}MA, Universidad de Concepción.

Entropy stable numerical schemes for divergence diminishing Chew, Goldberger & Low equations for plasma flows

Singh, Chetan; Kumar, Harish; Bhoriya, Deepak; Balsara, Dinshaw S.

Indian Institute of Technology Delhi, India

Tuesday, May 26, 17:15-17:40, M17.25 – 2.017

Chew, Goldberger & Low (CGL) equations are a set of hyperbolic PDEs with non-conservative products used to model the plasma flows, when the assumption of *local thermodynamic equilibrium* is not valid, and the pressure tensor is assumed to be rotated by the magnetic field. This results in the pressure tensor, which is described by the two scalar components. As the magnetic field also evolves, controlling the divergence of the magnetic field is important. In this work, we consider the generalized Lagrange multiplier (GLM) technique for the CGL model. The resulting model is referred to as the GLM-CGL system. To make the system suitable for entropy-stable schemes, we reformulate the GLM-CGL system by treating some conservative terms as non-conservative. The resulting system has a non-conservative part that does not affect entropy evolution. We then propose entropy stable numerical methods for the GLM-CGL model. The numerical results for the GLM-CGL system are then compared with the CGL system without the GLM divergence diminishing approach to demonstrate that the GLM approach indeed leads to significant improvement in the magnetic field divergence diminishing.

Quantification of numerical entropy loss for ocean models with a projection method

Boussard, Marie; Aguillon, Nina; Salomon, Julien

Inria Paris, Sorbonne Université, France

Tuesday, May 26, 17:40-18:05, M17.25 – 2.017

Finite-Volume schemes used to approximate the hyperbolic part of ocean circulation models exhibit a significant loss of numerical entropy. This effect is amplified by the coarse spatial resolution - on the order of 10^4 km^2 - resulting from the planetary space scale and from realistic computational constraints. It leads to unphysical behaviours, such as the mixing of water masses with distinct temperatures and salinities. To address this problem, referred to as "spurious mixing" by oceanographers, localising and quantifying this numerical entropy loss in high-order codes is crucial. A method was proposed by Aguillon, Audusse, Desveaux, and Salomon [2], but is limited to the one-dimensional case.

We propose a method of quantification applicable to two-dimensional high-order finite-volume schemes, suitable for unstructured meshes, in presence of source terms. The main idea is to project a consistent numerical entropy flux, like the centred flux, on the set of fluxes satisfying a discrete entropy inequality. This projection is carried out by solving an optimisation problem associated with inequality constraints. Properties of the projected flux will be discussed. In particular, under realistic assumptions, it can serve to establish - via an extended Lax-Wendroff theorem - that if convergence occurs, it is towards the entropic solution. Numerical examples will also be provided.

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P6-5 Structure Preserving Methods

A second-order invariant-region-preserving scheme for a transport-flow model of polydisperse sedimentation

Bürger, Raimund; Barajas-Calonge, Juan; Mulet, Pep; Villada, Luis Miguel

Universidad de Concepción, Chile

Thursday, May 28, 14:00-14:25, M17.25 – 2.017

A polydisperse suspension is a mixture of a number N of species of small solid particles, which may differ in size or density, dispersed in viscous fluid. The sedimentation of such a mixture gives rise to the segregation of species and flow of the mixture due to density fluctuations. In two space dimensions, and for equal-density particles, this process can be described by a hyperbolic system of N nonlinear conservation laws for the particle volume fractions coupled with a version of the Stokes system for the volume-averaged flow field of the mixture. A second-order numerical scheme for this transport-flow model is formulated by combining a finite-difference approximation of the Stokes system with a finite volume (FV) scheme for the transport equations, both defined on a Cartesian grid on a rectangular domain. The FV scheme is based on a central weighted essentially non-oscillatory (CWENO) reconstruction [M. J. Castro and M. Semplice, Int. J. Numer. Methods Fluids, 89 (2019), pp. 304–325] applied to the first-order local Lax-Friedrichs (LLF) numerical flux. By the application of scaling limiters to the CWENO reconstruction polynomials (following [X. Zhang and C.-W. Shu, J. Comput. Phys., 229 (2010), pp. 3091–3120]) and utilizing that the Stokes solver generates a discretely divergence-free (DDF) velocity field, one can prove that the FV scheme has the invariant region preserving (IRP) property, i.e., the volume fractions are nonnegative and sum up at most to a set maximum value. Numerical examples illustrate the model and the scheme.

A positivity-preserving segregated Runge-Kutta finite volume scheme for compressible flows on staggered meshes.

Harbreteau Thomas; Herbin Raphaèle ; Latché Jean-CLaude

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Thursday, May 28, 14:25-14:50, M17.25 – 2.017

In this work, we present a positivity-preserving quasi-second order in space and time finite volumes scheme on staggered meshes for Euler equations with (or without) added diffusion. This is an intermediate step in developping a compressible Navier-Stokes solver on staggered grids with the same properties, which is to be used in a LES method by the research team at Autorité de Sûreté Nucléaire et de Radioprotection (ASNR).

The time-stepping strategy is a two-stages Segregated Runge-Kutta (SRK) method. It is positivity preserving for the density and internal energy thanks to a flux limitation in-between intermediate Runge-Kutta steps derived from [3]. The limiter used is the Algebraic-MUSCL (A-MUSCL) algorithm [1]. It computes quasi-second order positivity preserving inter-cell values by projecting high-order (not positivity preserving) inter-cell approximations on a stability interval given by the values of adjacent cells. It is an easy-to-implement limiter that works well regardless of the space dimension.

The space discretization is staggered. Becaus of this, the internal energy balance is discretized instead of the total energy balance. A Lax-Wendroff consistency analysis of the total energy equation (the sum of the internal energy balance and a kinetic energy balance derived from the momentum balance equation) allows us to compute corrective terms that are added as source

terms to the internal energy balance so that the scheme computes correct shock solutions in the inviscid case.

An on-going work is to make the scheme more efficient by allowing the time-steps to be larger than in standard SRK methods.

[1] Piar, Libuse and Babik, Fabrice and Herbin, Raphaele and Latché, Jean-Claude A formally second order cell centered scheme for convection-diffusion equations on unstructured non-conforming grids *International Journal for Numerical Methods in Fluids*

[2] Gastaldo, L. and Herbin, R. and Latché, J.-C. and Therme, N. A MUSCL-type segregated – explicit staggered scheme for the Euler equations *Computers and Fluids*

[3] Ern, Alexandre and Guermond, Jean-Luc Invariant-domain-preserving high-order time stepping: II. IMEX Schemes *SIAM Journal on Scientific Computing*

Physical constraint preserving alternative finite-difference WENO scheme for hyperbolic systems with non-conservative products

Bhoriya, Deepak; Balsara, Dinshaw; Chandrashekar, Praveen and Shu, Chi-Wang

Birla Institute of Technology and Science Pilani, India

Thursday, May 28, 14:50-15:15, M17.25 – 2.017

Finite difference Weighted Essentially Non-Oscillatory (WENO) schemes fill an important need in computational science – in multi-dimensions, they can carry out high accuracy simulations at a fraction of the cost of finite volume and discontinuous Galerkin alternatives. Alternative Finite Difference WENO (AFD-WENO) schemes are variants of WENO schemes that provide the added flexibility of handling structured meshes with complex geometry. AFD-WENO schemes have also been designed recently to handle hyperbolic PDEs with non-conservative products (Balsara et al., Efficient Alternative Finite Difference WENO Schemes for Hyperbolic Systems with Non-Conservative Products, CAMC), thereby dramatically increasing the range of application areas to which they can be applied. However, one of the deficiencies of higher order WENO schemes stems from the fact that there are stringent problems where the schemes are not Physical Condition Preserving (PCP).

In this work, we rectify the above-mentioned deficiency by designing and documenting AFD-WENO schemes with a PCP property. The essential idea is to selectively hybridize a low-order scheme, which is PCP, with a high-order scheme, which may sometimes lose the PCP property. These ideas are packaged into a formulation that works with the fluctuation form for hyperbolic systems, and yet, our formulation is as easy to implement as PCP conditions for conservation laws. The formulation can work seamlessly for conservation laws as well as PDEs with non-conservative products. When the PDE has a conservation form in some limits, the formulation is designed to retrieve that conservation form. We numerically observe that the formulation does not damage the high order accuracy for problems with smooth flow. Several extremely stringent test problems are presented to illustrate the value of the PCP methods developed here.

A kinetic energy preserving discontinuous Galerkin scheme based on a flexible velocity kinetic model

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Thursday, May 28, 15:15-15:40, M17.25 – 2.017

The correct evolution of kinetic energy is desirable in simulation of compressible turbulent flows. Higher order accurate numerical methods such as flux differencing based nodal discontinuous Galerkin (DG) methods depend on kinetic energy preserving (KEP) numerical fluxes to ensure a faithful evolution of kinetic energy [1]. In this work, we present a new KEP numerical flux for the compressible Euler equations derived from the flexible velocity kinetic model of [2] for the Boltzmann equation.
Since the Euler equations can be recovered from the kinetic model by taking suitable moments, the new flux is constructed to ensure that the finite volume scheme for the kinetic model yields a KEP discretization of the Euler equations after the moment operation. By coupling the proposed KEP flux with the higher order DG framework, we demonstrate accuracy, robustness, kinetic energy preservation and pressure equilibrium preservation property of the resulting scheme through a series of benchmark problems.

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P7-1 Kinetic and Moment Equations

A semi-Lagrangian method for the polyatomic ESBGK model

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Monday, May 25, 14:00-14:25, M17.74 – 7.017

Polyatomic kinetic models are essential for accurately capturing the thermodynamic behavior of real gases, as internal energy modes significantly influence transport coefficients, relaxation processes, and non-equilibrium effects that cannot be represented by monoatomic models. The polyatomic ESBGK model [3] describes molecular collisions as a relaxation towards a generalized Gaussian distribution with an anisotropic covariance matrix and an exponentially decaying internal energy distribution. We present a new semi-Lagrangian scheme for the polyatomic Ellipsoidal Statistical BGK (ESBGK) model of the Boltzmann equation.

The semi-Lagrangian framework [2], being deterministic and grid-based, removes the time-step restriction associated with the linear transport term by following the method of characteristics. The potentially stiff relaxation term is treated using an implicit A-stable linear multistep method which, owing to the structure of the BGK operator, can be reformulated into a fully explicit time-stepping scheme. This yields a highly efficient and numerically stable method. Ultimately, the method is second-order in time, third-order in space, and spectral in the microscopic velocity. The numerical method is asymptotic preserving and stiffly accurate, meaning the scheme asymptotically converges to a scheme for the Euler equations in the vanishing Knudsen limit. In addition, we prove that the first-order scheme, asymptotically converges to the compressible Navier-Stokes equation. Finally, we propose inflow and outflow boundary conditions suitable for BGK-type kinetic equations. We perform simulations of the Fourier and Couette test case [1] to compare the BGK model with Direct Simulation Monte Carlo (DSMC). In addition, we perform simulations of an orifice geometry.

Acknowledgements. The authors have received funding from the European Union's Framework Program for Research and Innovation HORIZON-MSCA-2021-DN-01 under the Marie Sklodowska-Curie Grant Agreement Project 101072546 – DATAHYKING.

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Optimal convergence estimate of the limit from inverse power potential to hard sphere Boltzmann equation

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Monday, May 25, 14:25-14:50, M17.74 – 7.017

The inverse power potential $U(r) = r^{-1/s}, 0 < s < 1$, generates the Boltzmann kernel $B^s = |v - v_*|^{1-4s} b_s(\theta)$ with an angular singularity as $\theta \to 0$. Jang-Kepka-Nota-Velázquez ["Vanishing angular singularity limit to the hardsphere Boltzmann equation", J. Stat. Phys., 190(4), 2023] proved the limit $B^s \to \frac{1}{4}|v - v_*|$ as $s \to 0$, as well as weak convergence of solutions, but without a rate. In this work we establish the sharp estimate

$$|b_s(\theta) - \tfrac{1}{4}| \leq C\, s\, \theta^{-2-2s}.$$

In particular, this sharp estimate yields the *optimal* $O(s)$ convergence rate for solutions of the homogeneous Boltzmann equation with large initial data in suitable Sobolev spaces; i.e., for any $t \in [0, T]$, we have

$$f^s(t) = f^0(t) + O(s),$$

quantified by the L^1_k norm for $k \geq 2$.

Build, analyse and simulate hierarchical PDEs for free-surface flows with Zoomy

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Monday, May 25, 14:50-15:15, M17.74 – 7.017

Hierarchical partial differential equations (PDEs) enable the description of physical systems with variable model fidelity. Built using moment-based expansions with a variable set of polynomial basis functions to reduce the dimensionality of the system, such hierarchical models offer flexibility and computational efficiency. However, their development—particularly the analytical derivation of fluxes, eigenstructures, or boundary conditions—becomes increasingly complex and often infeasible to perform manually. This complexity, combined with the typical disconnect between symbolic derivation and high-performance numerical simulation, hinders rapid prototyping for complex real-world test cases.

Here, we present *Zoomy*, a novel framework that bridges this gap. It employs a *symbolic layer* to define an analytical system and automatically generates high-performance numerical kernels for systems of PDEs, which are then forwarded to a *numerical layer*. We demonstrate that flux functions, source terms, eigenstructures, and boundary conditions can be generated and solved in an automated fashion, enabling the immediate simulation of complex 1D or 2D test cases. The numerical backend is easily exchangeable; we showcase finite-volume solvers based on *NumPy*, *JAX*, and *AMReX*, as well as a discontinuous Galerkin solver based on *Firedrake*. We validate the framework using hierarchical *Shallow Moment Equations* and *Vertically Averaged and Moment Equations*, demonstrating that *Zoomy* enables the effortless exploration of model hierarchies that would otherwise be cumbersome to implement. This symbolic-to-numeric approach drastically accelerates the model development cycle.

A model-adaptive discontinuous Galerkin scheme for linearized hierarchical moment equations

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Monday, May 25, 15:15-15:40, M17.74 – 7.017

Moment equations are a promising approach to model gas flows at microscopic scales, where the classical Navier-Stokes-Fourier equations fail to accurately predict important flow quantities. By selecting different cutoff points in the Grad-moment-expansion of the Boltzmann equation, which can be understood as a Galerkin projection in velocity space, a nested model hierarchy consisting of moment equations is constructed (see [2]). When linearized and augmented with suitable boundary conditions, these systems of equations are hyperbolic and well-posed. Since the computational cost scales with the level of the model inside the hierarchy, it suggests itself to exploit anisotropy in the solution by locally adapting the model-level, such that an overall target accuracy is achieved more efficiently compared to the case of a single global model. In this work, we present an element-wise model-adaptive discontinuous Galerkin discretization on unstructured quadrilateral meshes. Neighboring elements with different model-levels are coupled via two half-Riemann problems [2] embedded at each quadrature node of shared interfaces, thereby enforcing a set of coupling conditions. Various coupling conditions are derived and their influence on the numerical solution and the energy stability of the discretization is investigated. The resulting scheme is embedded into a solve-estimate-mark-refine loop, where model-refined solutions are used to estimate the local model error. The efficacy of the present approach with respect to the needed number of degrees of freedom and runtime is evaluated by simulating a flow through a microscopic channel with curved obstacles. While the results show promise in reducing computational cost in settings where high model fidelity is required, further work on the coupling conditions and error estimation is required to increase the accuracy and robustness of the scheme.

Acknowledgements. This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 333849990/GRK2379 (IRTG: Hierarchical and Hybrid Approaches in Modern Inverse Problems).

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P7-2 Kinetic and Moment Equations

A moment based extension to the shallow water equations

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Monday, May 25, 16:00-16:25, M17.74 – 7.017

The well-known shallow water equations form a very simple system and, despite this simplicity, have a wide range of applications. However, they have the significant disadvantage that information about the vertical velocity profile is lost due to vertical averaging, and therefore they cannot be used to describe processes in which this direction plays a non-negligible role. To overcome this problem, the shallow moment equations were introduced. They allow us to retain information about the vertical direction while incurring lower computational costs compared to the incompressible free-surface Euler equations. They lie between the shallow water equations and the incompressible free-surface Euler equations and can be regarded as a model hierarchy with both systems as limiting cases.

It is assumed that the velocity profile in the vertical direction can be sufficiently approximated by scaled and normalized Legendre polynomials, due to their orthogonality properties. We present the essential steps required to derive the shallow moment equations from the incompressible free-surface Euler equations using this approach and identify the point at which global hyperbolicity breaks down. In the case $N = 0$, we recover the classical shallow water equations, which serve as the starting point of the model hierarchy for increasing levels N.

We present the fully nonlinear shallow moment model in the case where the pressure consists not only of a hydrostatic part but also includes a non-hydrostatic component. The incorporation of non-hydrostatic effects allows the model to capture more complex flow phenomena, such as dispersive wave propagation and vertical accelerations, which cannot be described within a purely hydrostatic framework. For the pressure deviation, we apply the same moment approach as for the velocity. Even though this system lacks global hyperbolicity, it satisfies an energy conservation law. We demonstrate how this energy equation is derived from the general system in the two-dimensional case and highlight the crucial steps.

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Moment approximations to magnetic rotating shallow flows

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Monday, May 25, 16:25-16:50, M17.74 – 7.017

Originally introduced to describe a transition region in stars, the magnetic rotating shallow water (MRSW) model is now used in many solar physics and geophysical applications. Derived from the 3-D incompressible magnetohydrodynamic system, the shallow nature of these applications motivates depth-averaging of both the velocities and magnetic fields. This is advantageous in terms of computational efficiency—but at the loss of vertical information, thus limiting the predictive power of the MRSW model. To overcome this problem, we employ higher-order vertical moments, but now in the context of conductive fluids. In doing so, the new approximation maintains non-constant vertical profiles of both the horizontal magnetic fields and horizontal velocities, while still remaining in the simplified 2-D framework corresponding to depth integration. In this work, we extend the derivation of the shallow water moment equations to derive

the MRSW moment system of arbitrary order; i.e., we represent the vertical profiles of the velocities—and now additionally the magnetic fields—by arbitrary-order polynomial expansions, and close the new expanded 2-D system with evolution equations for these polynomial coefficients, found via Galerkin projection. Through numerical experiments for MRSW moment systems up to third-order, we demonstrate that these moment approximations reduce model error without significantly sacrificing computational efficiency.

Beyond the log law: Legendre polynomial velocity profiles for shear stress and sediment transport in shallow water models

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Monday, May 25, 16:50-17:15, M17.74 – 7.017

The Shallow Water Equations (SWE) provide a hyperbolic model for free–surface flows, but they retain only a depth–averaged velocity and therefore do not represent vertical shear dynamically. Shallow Water Moment models (SWM) address this limitation by evolving a small set of vertical moments as additional state variables within an extended hyperbolic system. For real–world applications, for example in sediment transport problems, the key difficulty is modelling the friction term. Standard SWE closures for bed friction and sediment flux are commonly derived and calibrated under fully rough, equilibrium log–law assumptions. Once the velocity profile is allowed to evolve away from that equilibrium, these closures are no longer consistent with the resulting momentum balance, and the resulting bed shear/friction and momentum balance become sensitive to the nature of the perturbation. This motivates the central question of the present work: how should friction and sediment flux be formulated in a moment framework, where bed shear and shear within the water column emerge from the evolving moments rather than being imposed by an assumed velocity profile?

In this work, we introduce a vertical velocity representation based on shifted Legendre polynomials in a normalized depth coordinate, with a specific focus on friction and sediment– transport modelling. We first compute friction coefficients and bed–shear stress using the classical log–law velocity profile as a reference and show that it can be accurately reproduced by projecting the log–law onto a small number of Legendre modes. The key advantage is that both bed shear and the internal shear distribution are recovered directly from the moment coefficients, rather than being imposed a priori from an assumed profile. Furthermore, we show that the Legendre modes can capture a wide range of deviations from the logarithmic velocity profile. This provides, to our knowledge, the first moment–consistent formulation of bed shear stress for shallow water moment models, in which friction and sediment transport closures are directly linked to the evolving vertical velocity structure. The approach thus provides a natural foundation for shallow water moment models for sediment transport, in which a small number of vertical moments replaces the single depth–averaged velocity, improving the link between vertical flow structure, bed shear, and sediment fluxes without conducting a fully three– dimensional simulation. We present numerical results comparing the evolution of the free surface and velocity profiles obtained with a classical log–law-based shallow water model and with a Legendre polynomial–based shallow water moment model, where the bed shear stress is computed directly from the projected moment coefficients and shear within the water column is modelled using a Newtonian formulation with an eddy–viscosity–based effective kinematic viscosity.

A hat function-based positive contact discontinuity capturing Boltzmann scheme

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Monday, May 25, 17:15-17:40, M17.74 – 7.017

The suitable moments of the Maxwell velocity distribution function from kinetic theory of gases recover the conserved variable and flux vectors of the Euler equations of gas dynamics. The Maxwellian has infinite support in the velocity space, which poses challenges for numerical analysis of kinetic numerical methods. Although the natural choice for the velocity distribution function is Maxwellian, other choices that preserve the key moments often lead to more efficient kinetic schemes. In his seminal paper, Perthame [1] introduced a compactly supported *hat function* as an alternative to Maxwellian. This talk presents the formulation and analysis of a contact discontinuity capturing Boltzmann scheme based on peculiar velocity, constructed using the hat function. Owing to the compact support in velocity space of the hat function, the numerical analysis of the proposed Boltzmann scheme is considerably simplified, allowing for a direct proof of its positivity.
Piecewise linear reconstruction of the conserved variable vector is assumed in each cell for second-order spatial accuracy. However, such reconstruction may yield unphysical interface states, including negative density and pressure, which - since the equilibrium distribution function depends on density, velocity and pressure - lead to a loss of positivity of the resulting interface distribution function. To address this issue, we adopt a two-stage limiting strategy: the Venkatakrishnan limiter is applied first to reduce spurious oscillations, followed by the Zhang-Shu limiter, which enforces positivity preservation of density and pressure, and thus, resulting distribution function. As a result, positivity of the Euler forward-in-time, second-order in space contact discontinuity capturing Boltzmann scheme can be established subject to a CFL-like condition, and this property is preserved under strong stability preserving high order Runge-Kutta time discretizations. The proposed second-order scheme is tested on several benchmark test cases for inviscid flows in one- and two- dimensions.

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P7-3 Kinetic and Moment Equations

H-theorems for dense reactive mixtures and global existence of solutions

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Tuesday, May 26, 14:00-14:25, M17.74 – 7.017

Simple reacting spheres (SRS) model has been developed in [1] and further advanced in [2], [3], and [4], In the SRS model, the molecules behave as if they were single mass points with two internal states of excitation. Collisions may alter the internal states. I consider a dense mixture of four species A, B, A^*, B^*, consisting of hard spheres with diameters d_i and masses m_i, where indices $1, 2, 3, 4$ are for the particles A, B, A^*, B^*, respectively. In contrast to the dilute gas regime, in this work the two-particle distribution function of the mixtures, $f_{ij}^{(2)}$ is approximated (see, H. van Beijeren and M. H.Ernst, *The modified Enskog equation for mixtures*, Physica **70** (1973), 225-242) **not** by $f_{ij}^{(2)} = f_i f_j$ but by $f_{ij}^{(2)} = g_{ij}^{(2)} f_i f_j$, with f_i being a one-particle distribution function of the mixtures and $g_{ij}^{(2)}$ being the pair correlation function of hard spheres at non-uniform equilibrium, with well known algebraic expression. I assume that when a hard-sphere A collides with another hard-sphere B, there is a probability $0 \leq \alpha_R \leq 1$ where in the reaction $A + B \to A^* + B^*$, A^* and B^* are formed. Furthermore, m_i and d_i denote the mass and the diameter of the i-th particle, $i = 1, \ldots, 4$, and reactions take place when the reactive particles are separated by a distance $\sigma_{12} = \frac{1}{2}(d_1 + d_2) = \sigma_{34} = \frac{1}{2}(d_3 + d_4)$ and the kinetic energy associated with the relative motion exceeds the activation energies γ_i: $(1/2)\mu_{is}\big(\langle \epsilon, v - w\rangle\big)^2 \geq \gamma_i$, where $\mu_{is} = \frac{m_i m_s}{m_i + m_s}$ are the reduced masses. The conservation of mass has the form $m_1 + m_2 = m_3 + m_4$, where m_i denotes the mass of the i-th particle, $i = 1, \ldots, 4$. The nonlinear kinetic system is

$$\frac{\partial f_i}{\partial t} + v\frac{\partial f_i}{\partial x} = J_i^E + J_i^R, \quad i = 1, 2, 3, 4, \tag{9}$$

where J_i^E are elastic (hard-sphere) collisional operators and J_i^R are reactive collisional terms. The convergence of the infinite Mayer virial expansion for $g_{ij}^{(2)}$ is not known at present, instead, I consider a truncated series for $g_{ij}^{(2)}$, with finite, but arbitrary number of terms.

First, I show existence of an H-functional for system (9). This is the first such result for **dense reacting** systems. The H-functional: $H(t) = H_B(t) + H_{corr}(t)$ consists of two parts: the kinetic part, $H_B(t) = \sum_{i=1}^{4} \iint f_i \log f_i \, dv dx$, and the correlational part, $H_{corr}(t)$. The structure of $H_{corr}(t)$ is similar to the algebraic structure of the equilibrium (non-uniform) entropy for the mixture of hard–spheres. In addition, one can identify that $-\beta H_{corr} = \mathcal{A}^{excess}$ with $\beta = 1/kT$, where $\mathcal{A}^{excess}$ is the excess free energy for the mixture hard-spheres in a **non-uniform** equilibrium.

Next, I construct approximate solutions to (9). Starting with L^1 initial data, I utilize the conservation laws and the H-function to show convergence of the approximate solutions to a solution of (9). This is the first such existence result for **dense reactive** mixture.

Additionally, when $\alpha_R = 0$ (no reactive terms), system (9) becomes an inert dense mixture of hard spheres (in this case one can consider any number of species with arbitrary masses) and this work also provides the first global in time existence of solutions for such system.

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Gramian moment closures for kinetic equations: hyperbolicity and numerics

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Tuesday, May 26, 14:25-14:50, M17.74 – 7.017

Moment methods provide a systematic reduction of kinetic equations, such as the Boltzmann equation, to finite-dimensional systems of partial differential equations for macroscopic quantities [1, 2]. The primary difficulty lies in the closure problem: the evolution of a finite set of moments depends on higher-order moments, which must be approximated using lower-order moments. A physically admissible closure should preserve key structural properties, particularly hyperbolicity, to ensure local well-posedness.

A nonlinear closure based on orthogonal polynomials derived from the Gram matrix of velocity moments has been proposed for one-dimensional kinetic equations [3]. This Gramian closure yields explicit formulas for higher-order moments without reconstructing the underlying distribution function. An extended formulation introduces a free parameter that provides additional control over hyperbolicity and equilibrium preservation. We rigorously prove that the one-dimensional Gramian closure is hyperbolic for all realizable moment vectors, providing complete theoretical guarantees. Numerical experiments demonstrate that Gramian closures deliver accurate and robust predictions while maintaining computational efficiency. Comparisons with classical closures, including Grad-type and entropy-based methods, reveal advantages in structural stability and efficiency.

This talk presents the Gramian closure method, with emphasis on the hyperbolicity proof in one dimension and systematic comparison with classical moment closures. Extensions to multidimensional systems will also be discussed.

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Kinetic equations for quantum gases - consistency and long-time behavior

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Tuesday, May 26, 14:50-15:15, M17.74 – 7.017

Gas flows in physics are modeled either at space and timescales that we live in (macroscopic) or at the atomistic level (microscopic). Boltzmann introduced an intermediate scale, the kinetic description. In this talk, we focus on phenomena that are best described kinetically. I will give an overview of several quantum kinetic models, including the quantum Boltzmann equation, BGK-type relaxation models, and quantum Fokker–Planck equations. For these models we examine central questions both on the modeling side, such as suitable extensions to gas mixtures, and on the analytical side, in particular the exponential convergence to equilibrium for the non-linear equations.

A particular focus lies on proving consistency of the extended models, such as conservation of

mass, momentum, and energy, as well as the validity of an H-theorem leading to Fermi–Dirac or Bose–Einstein equilibria.
On the analytical side, I will highlight recent results concerning the long-time behavior of solutions, in particular exponential convergence to equilibrium for several nonlinear models. These results rely on hypocoercivity techniques adapted to the quantum context.

This is joint work with Gi-Chan Bae, Seok-Bae Yun, Christian Klingenberg, Sandra Warnecke, Gayrat Toshpulatov.

[1] G. Bae, C. Klingenberg, M. Pirner, S. Yun, BGK model of the multi-species Uehling Uhlenbeck equation, Kinetic and Related Models (2021) 14 (1): 25-44

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[4] M. Pirner, A consistent kinetic Fokker-Planck model for gas mixtures, Journal of Statistical Physics, 191, 143 (2024)

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Equilibrium boundary conditions for vectorial multi-dimensional lattice Boltzmann schemes

Bellotti, Thomas; Denise Aregba-Driollet

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Tuesday, May 26, 15:15-15:40, M17.74 – 7.017

The concept of equilibrium is a general tool to fill the gap between macroscopic and mesoscopic information, both within kinetic systems and kinetic schemes. This work, detailed in [1], explores the use of equilibria to devise numerical boundary conditions for multi-dimensional vectorial lattice Boltzmann schemes tackling systems of hyperbolic conservation laws. In the scalar case, we prove convergence for schemes with monotone relaxation to the weak entropy solution by Bardos, Leroux, and Nédelec [2], following the path by Crandall and Majda [3]. Numerical experiments are conducted both for scalar and vectorial problems (e.g., Figure 1), and demonstrate the effectiveness of equilibrium boundary conditions in capturing significant physical phenomena.

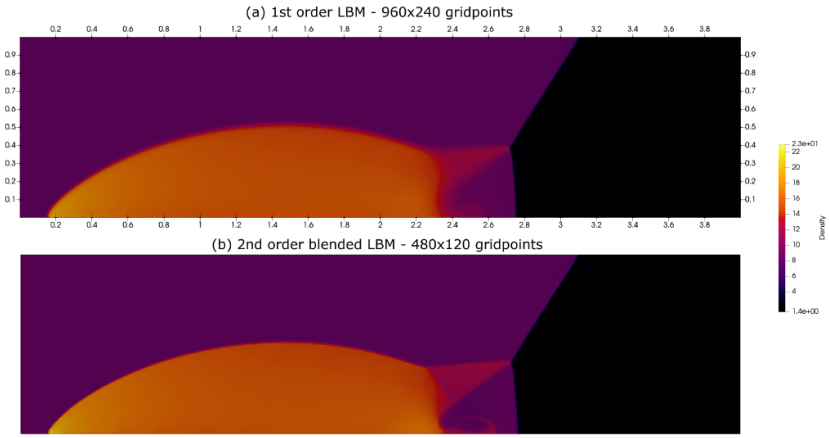

Figure 1: Example of computation on a double Mach 10 test case using boundary conditions built on equilibria with two different lattice Boltzmann schemes.

[1] Denise Aregba-Driollet and Thomas Bellotti. Equilibrium boundary conditions for vectorial multi-dimensional lattice Boltzmann schemes. Accepted in *Mathematics of Computation*, 2025.

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[3] Michael G. Crandall and Andrew Majda. Monotone difference approximations for scalar conservation laws. *Mathematics of Computation*, 34(149):1–21, 1980.

P7-4 Kinetic and Moment Equations

A kinetic framework for SIR-type epidemic models: mean-field limit and asymptotic analysis

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Tuesday, May 26, 16:00-16:25, M17.74 – 7.017

The goal of the talk is to provide a multi-scale perspective bridging kinetic theory with classical epidemic models. To this aim, we propose a kinetic framework to describe the time evolution of the statistical distributions of susceptible f_S, infectious f_I and recovered populations f_R, under epidemic transmission driven by susceptible-infectious interactions. The resulting formulation consists of a system of Boltzmann-like equations

$$\begin{aligned}
\frac{\partial f_S(x,t)}{\partial t} &= \mathcal{Q}_{SI}(f_S,f_I)(x,t) + \mathcal{R}^{\alpha}(f_S,f_R)(x,t),\\
\frac{\partial f_I(x,t)}{\partial t} &= \mathcal{Q}_{IS}(f_I,f_S)(x,t) + \mathcal{L}^{-\gamma}(f_I)(x,t),\\
\frac{\partial f_R(x,t)}{\partial t} &= \mathcal{R}^{\gamma}(f_I,f_R)(x,t) + \mathcal{L}^{-\alpha}(f_R)(x,t),
\end{aligned}$$

where $x,t \geq 0$. Bilinear terms $\mathcal{Q}_{SI}$ and $\mathcal{Q}_{IS}$ model the binary interactions between susceptible and infectious individuals, while recovery and reinfection dynamics are taken into account by linear redistribution operators $\mathcal{R}^{\gamma}, \mathcal{L}^{-\gamma}$ (γ being the recovery rate) and $\mathcal{R}^{\alpha}, \mathcal{L}^{-\alpha}$ (α being the loss of immunity rate), respectively.

At the macroscopic scale, the first order moments of the kinetic system are governed by an SIR-type model with reinfection, whose parameters are rigorously derived from the underlying microscopic interaction rules. Moreover, in the grazing collision regime, the Boltzmann system is properly approximated by a system of Fokker–Planck equations

$$\begin{aligned}
\frac{\partial f_S(x,t)}{\partial t} &= \frac{\sigma_S}{2} m_I(t) \frac{\partial^2}{\partial x^2}\left[x^2 f_S(x,t)\right] + \frac{\partial}{\partial x}\left[\left(\beta m_I(t)x - \alpha m_R(t)\right) f_S(x,t)\right],\\
\frac{\partial f_I(x,t)}{\partial t} &= \frac{\sigma_I}{2} m_S(t) \frac{\partial^2}{\partial x^2}\left[x^2 f_I(x,t)\right] + \frac{\partial}{\partial x}\left[\left(\left(\gamma\theta - \beta m_S(t)\right)x - \gamma(\theta-1)m_I\right) f_I(x,t)\right],\\
\frac{\partial f_R(x,t)}{\partial t} &= \frac{\sigma_R(\alpha)}{2} \frac{\partial^2}{\partial x^2}\left[x^2 f_R(x,t)\right] + \frac{\partial}{\partial x}\left[\left(\alpha x - \gamma m_I(t)\right) f_R(x,t)\right],
\end{aligned}$$

yielding a more tractable and detailed analysis of the long-time behaviour of the population distributions, with or without reinfetion. Within this setting, we can rigorously prove convergence to equilibrium of the mean-field system through the energy distance method

$$\begin{aligned}
\mathcal{E}^p(f,f_\infty) &= 2\int_{\mathbb{R}_+\times\mathbb{R}_+} |x-y|^{2p-1} f(x) f_\infty(y)\,dxdy - \int_{\mathbb{R}_+\times\mathbb{R}_+} |x-y|^{2p-1} f(x) f(y)\,dxdy\\
&\quad - \int_{\mathbb{R}_+\times\mathbb{R}_+} |x-y|^{2p-1} f_\infty(x) f_\infty(y)\,dxdy = \frac{2p-1}{\sqrt{2}} \frac{2^{2p-1}\Gamma(p)}{\Gamma\left(\frac{3}{2}-p\right)} \int_{\mathbb{R}} \frac{|\hat{f}(\xi,t) - \hat{f}_\infty(\xi)|^2}{|\xi|^{2p}},
\end{aligned}$$

with $1/2 < p < 3/2$. The analysis also highlights the dissipative nature of the dynamics and clarifies the role of interaction terms in driving the system toward a stable equilibrium.

Some numerical tests are proposed to highlight the consistency of the approach and the model is used to show its capability to accurately reproduce the COVID-19 epidemiological data observed in Italy.

This work is a collaboration with G. Toscani, R. Travaglini and M. Zanella.

Hydrodynamic limit of the Boltzmann equation towards Riemann solutions composed of shocks

Choe, Mingi; Kang, Moon-Jin; Kim, Chanwoo

KAIST, South Korea

Tuesday, May 26, 16:25-16:50, M17.74 – 7.017

The hydrodynamic limit of the Boltzmann equation towards shock solution of the compressible Euler equations is a challenging problem in kinetic theory. We study the hydrodynamic limit of the one-dimensional Boltzmann equation with hard-sphere collisions towards generic Riemann solutions composed of shocks like superposition composed of shock, contact discontinuity and rarefaction waves, or of two shocks and contact discontinuity. We prove the convergence in L^2 globally in time and space. The proof is based on the method of a-contraction with shifts, and the vanishing property of layers in the limit. This work is based on joint work with Moon-Jin Kang (KAIST) and Chanwoo Kim (UW-Madison).

Long-time asymptotic analysis of the Vlasov equation under a strong magnetic field

Trullo, Manuela; Filbet, Francis; Rodrigues, Miguel

University of Ferrara, Italy

Tuesday, May 26, 16:50-17:15, M17.74 – 7.017

We study the long-time asymptotic behavior of solutions to the Vlasov equation in the presence of a strong external magnetic field. We provide a mathematically rigorous derivation of the guiding-center approximation in a general three-dimensional setting with large inhomogeneous magnetic fields. First-order corrections are computed and justified, including electric cross-field, magnetic gradient, and curvature drifts.

The analysis is entirely carried out in a long-time regime, highlighting how these drift effects accumulate and determine the effective dynamics. We illustrate the results on two specific examples: the two-dimensional case in cartesian coordinates and a toroidal axisymmetric geometry. The approach relies on algebraic manipulations that exploit the structure of the stiffest part of the system of characteristics, including linear, quadratic, and trilinear contributions.

[1] F. Filbet and L. M. Rodrigues. Asymptotically preserving particle methods for strongly magnetized plasmas in a torus. *J. Comput. Phys.*, 480:Paper No. 112015, 23, 2023. doi:10.1016/j.jcp.2023.112015

[2] F. Filbet and L. M. Rodrigues. Asymptotics of the three-dimensional Vlasov equation in the large magnetic field limit. *J. Éc. polytech. Math.*, 7:1009–1067, 2020. doi:10.5802/jep.134

P8-1 Traffic Flow

Nonlocal models with time delay for mixed autonomous/human-driven traffic

Ciaramaglia, Ilaria

Inria Centre d'Université Côte d'Azur, France

Sapienza University of Rome, Italy

Monday, May 25, 14:00-14:25, M17.52 – 5.019

Traffic flow is a complex phenomenon that deeply impacts our daily lives, the economy and the environment. Studying and understanding traffic behavior is crucial to develop solutions that can alleviate congestion and improve road safety.
We aim to investigate heterogeneity in traffic by introducing some mathematical models which are able to study the interactions between human drivers and autonomous vehicles (AVs). One of the potential benefits of an intelligent transportation system (ITS) regards safety: AVs can dramatically reduce accidents caused by human error, with an estimated fatal crash-rate reduction of at least 40%. In addition, AV technology has the potential to improve traffic flow and reduce congestion and fuel consumption. This is due to the fact that AVs could enable quicker reaction times and closer spacing between vehicles to counteract increasing demand, thus leading to smoother traffic patterns.

We propose some macroscopic models which are able to capture some specific features of human behavior on the road, including nonzero reaction time and short/long range interactions among vehicles. The models are nonlocal extensions to the widely-known Lighthill-Whitham and Richards (LWR) model, and therefore retain the structure of hyperbolic conservation laws. Other nonlocal extensions have been recently introduced in the literature to overcome some drawbacks of local dynamic descriptions. These approaches are based on the assumption that drivers adapt their speed to a weighted mean of the downstream traffic density or velocity. Moreover, many real-world traffic phenomena require a transient phase to become effective, due to humans' delayed physiological reaction to external stimuli. Following this idea, a delay is also incorporated in the models, thus taking into account for humans reaction time. Through these extensions, we intend to investigate the interactions among several classes of vehicles with different behaviors in terms of look-ahead distance and reaction time, with particular relevance to mixed traffic scenarios involving autonomous and human-driven vehicles (HVs). Indeed, we can assume that smart cars can collect information on the surrounding traffic within a large perimeter and are able to instantaneously respond to external events, whereas human drivers react to downstream traffic in a shorter range and with a nonzero time delay.

We provide well-posedness of entropy weak solutions. To prove existence, we construct finite-volume approximate solutions using a Hilliges-Weidlich scheme and we provide uniform $\mathbf{L}^\infty$ and $\mathbf{BV}$ estimates, thus ensuring convergence. Then, we prove that the limit function is indeed an entropy weak solution of the model. We also achieve an $\mathbf{L^1}$ stability result by adapting Kružkov's doubling of variables technique, from which both uniqueness and convergence to the associated non-delayed model follow. The latter also constitutes an improvement of the result obtained in the literature as in the current scenario we prove the convergence for any chosen time horizon. We propose some numerical tests, with a particular focus on the capability of AVs of introducing a stabilizing effect in a mixed AVs/HVs environment. Our analysis also investigates how the dimension of the group of AVs introduced into the traffic mixture affects their stabilizing capabilities. Moreover, several studies have shown that even a very small number of controlled vehicles can effectively regulate traffic, dampening stop-and-go waves and reducing fuel consumption as well as pollutant emissions. Hence, we further examine whether our models are consistent with these experimental findings.

Conservation laws with discontinuous gradient-dependent flux

Amadori, Debora; Bressan, Alberto; Shen, Wen

University of L'Aquila, Italy

Monday, May 25, 14:25-14:50, M17.52 – 5.019

We consider a scalar conservation law with discontinuous gradient-dependent flux, where the discontinuity is determined by the sign of the derivative u_x of the solution; namely, the flux is given by two different smooth functions $f(u)$ or $g(u)$, when u_x is positive or negative, respectively. This problem is motivated by traffic flow modeling, under the assumption that drivers exhibit different behaviors in accelerating or decelerating mode.

A vanishing viscosity approximation identifies two different situations according to the mutual positions of the graphs of f and g: a well-posed parabolic problem when $f(u) < g(u)$ for all u, or an ill-posed problem if $f(u) > g(u)$ for all u. We refer to these problems as the stable/unstable case respectively.

In the stable case [1], we prove that trajectories obtained as a limit of a suitable wave-front tracking algorithm coincide with the unique limits of vanishing viscosity approximations, and yield a contractive semigroup in L^1.

On the other hand, the unstable case [2] leads to non-uniqueness of the solutions, and has a possible interpretation in traffic flow, with the random emergence of stop-and-go waves as a result of the instability. An analysis in this direction is the object of the forthcoming paper [3].

[1] D. Amadori, A. Bressan and W. Shen. Conservation laws with discontinuous gradient-dependent flux: the stable case. *Math. Models Methods Appl. Sci.*, 35(6):1421–1469, 2025.

[2] D. Amadori, A. Bressan and W. Shen. Conservation laws with discontinuous gradient-dependent flux: the unstable case, arXiv:2411.13444, 2024

[3] A. Bressan and S. K. Suresha. A two-fluxes stochastic model of traffic waves. In preparation.

A two-fluxes stochastic model of traffic waves

Kanale Suresha, Sumantha; Bressan, Alberto

Pennsylvania State University, United States of America

Monday, May 25, 14:50-15:15, M17.52 – 5.019

This talk is concerned with a scalar conservation law with a discontinuous, gradient-dependent flux. Namely, we take the flux to be $f(u)$ if the gradient u_x is positive and $g(u)$ when the gradient is negative. Recent work by Amadori, Bressan and Shen has considered the stable case, where $f < g$ and the Cauchy problem is well posed, and the unstable case where $f > g$ and infinitely many piecewise monotone solutions can arise for the same initial data.

The present talk will examine more specifically the case where the graphs of f and g are both strictly concave, but with one intersection. This setting naturally arises in traffic flow models. Indeed, traffic data associated with hysteresis effects suggests that for low vehicle densities one should have $f < g$, while for higher densities $f > g$. This instability can produce spikes in the traffic density in response to arbitrarily small random perturbations, leading to stop-and-go waves. The talk will introduce a simple stochastic model, where the emergence of new spikes follows a Poisson distribution in space and time, and examine the change in the average speed and acceleration.

[1] D. Amadori, A. Bressan and W. Shen. Conservation laws with discontinuous gradient- dependent flux: the stable case. *Math. Models Methods Appl. Sci.* 35 (2025), 1421– 1469.

[2] D. Amadori, A. Bressan and W. Shen. Conservation laws with discontinuous gradient- dependent flux: the unstable case. *Comm. Math. Sci.* ,submitted.

[3] A. Bressan and S.K. Suresha, A two-fluxes stochastic model of traffic waves. Work in progress.

Adapting priority Riemann solver for GSOM on road networks

Briani, Maya; Balzotti, Caterina; Bianchini, Roberta; Piccoli, Benedetto

Istituto per le Applicazioni del Calcolo "M. Picone" - CNR, Italy

Monday, May 25, 15:15-15:40, M17.52 – 5.019

This presentation introduces an extension of Generic Second Order Models (GSOM) for traffic flow on complex road networks. We develop a Riemann solver for junctions characterized by n incoming and m outgoing roads, governed by an adaptive priority rule. The proposed solver maximizes flux while dynamically adjusting priorities when the supply of an outgoing road exceeds the demand of high-priority inflows. To ensure the practical applicability of the model, we provide an iterative algorithm for constructing junction solutions and use wave-front tracking to generate approximate solutions for Cauchy problems. A central contribution of this work is the rigorous bounding of total variation for waves interacting with the junction. We provide explicit calculations for 2×2 road configurations to demonstrate the model's behavior. The core novelty lies in our detailed analysis of returning waves—waves generated at the junction that return after interacting along the road segments. We demonstrate that, in contrast to first-order models like the Lighthill-Whitham-Richards (LWR), these interactions in a second-order framework can significantly increase flux variation. These findings provide a perspective on the influence of high-order dynamics on network stability.

P8-2 Traffic Flow

A nonlocal upstream-downstream traffic flow model

Villada-Osorio, Luis-Miguel

GIMNAP-Universidad del Bío-Bío, Concepción-Chile

Monday, May 25, 16:00-16:25, M17.52 – 5.019

In this talk we analyze the well-posedness of a new class of macroscopic vehicular traffic model, which is described by a scalar nonlocal conservation law that simultaneously incorporates both upstream and downstream effects in the flow dynamics. Unlike nonlocal models previously described in the literature, which only account for downstream density averages (look-ahead behavior), the proposed model introduces an additional term depending on an upstream average (look-behind), allowing for a more realistic representation of anticipatory driver behavior under high-density conditions. Inspired by the multiplicative flux proposed in [I. Karafyllis, D. Theodosis, and M. Papageorgiou. Analysis and control of a non-local pde traffic flow model. International Journal of Control, 95(3):660–678, 2022], our model generalizes and adapts such ideas to an entropy weak solution framework, allowing for the presence of discontinuities and shock waves. The considered flux takes the form $\rho\, g(\rho)\, W(\hat{R}_\delta)\, V(R_\eta)$, where the nonlocal terms $\hat{R}_\delta$ and R_η represent backward- and forward-looking spatial averages of the density, respectively, and the functions W and V encode the drivers' responses to these observations. The main novelty of this work lies in establishing the existence and uniqueness theory for entropy weak solutions, together with a rigorous proof of Lipschitz continuous dependence of solutions not only on the initial data, but also on the kernel functions, under reasonable structural assumptions on the flux components. The proofs are achieved through the design of a conservative numerical scheme that preserves key structural properties of the continuous model, such as maximum principle, mass conservation, $\mathbf{BV}$ estimates, and $\mathbf{L}^1$-stability. Finally, we present numerical experiments that illustrate the behavior of solutions and the qualitative impact of nonlocal terms on traffic dynamics.

This presentation is based on joint research with Harold Contreras (USS-Chile) and Paola Goatin (INRIA-Sophia-Antipolis, France).

[1] F. A. Chiarello, H. D. Contreras and L. M. Villada. Nonlocal reaction traffic flow model with on-off ramps. *Netw. Heterog. Media*, **17** (2) (2022), pp. 203–226.

[2] F. A. Chiarello, H. D. Contreras and L. M. Villada. Existence of entropy weak solutions for 1D non-local traffic models with space-discontinous flux. *J. Eng. Math.*, **141** (2023), article 9.

[3] H. D. Contreras, P. Goatin and L. M. Villada. A two-lane bidirectional nonlocal traffic model. *J. Math Anal. Appl.* **543** (2) (2025), article 129027.

[4] H. D. Contreras, P. Goatin and L. M. Villada. Well-posedness of a nonlocal upstream-downstream traffic model. *Submitted 2025.*

Micro to macro: from infinite follow the leader ODEs to general traffic models

Marcellini, Francesca; Colombo Rinaldo M., Guerra Graziano
University of Brescia, Italy

Monday, May 25, 16:25-16:50, M17.52 – 5.019

We consider the Cauchy problem for the following system of partial differential equations

$$\begin{cases} \partial_t \rho + \partial_x \big(\rho\, v(\rho, w)\big) = 0 \\ \partial_t w + v(\rho, w)\, \partial_x w = 0 \end{cases} \qquad (t, x) \in \mathbb{R}_+ \times \mathbb{R}\,, \tag{10}$$

which describes the movement, in one space dimension, of a population whose density $\rho = \rho(t, x)$ moves according to the velocity field $v = v(\rho, w)$. The vector quantity $w = w(t, x)$ acts as a *Lagrangian marker*, i.e., as a quantity attached to each *individual* composing the population. This system comprises a variety of traffic models, such as the one by Lighthill-Whitham and Richards; the 2×2 Aw-Rascle and Zhang system; its GARZ, or GSOM, generalization; the 2-Phase model [3] as well as further extensions.

Derivations of particular cases of (10) from microscopic models are available in the literature. The usual starting point is the Follow-the-Leader model, where the traffic dynamics is governed by the interaction between a vehicle, say sited at x_i, and the vehicle immediately in front, say at x_{i+1}:

$$\dot{x}_i = v\left(\frac{\ell}{x_{i+1} - x_i}, w_i\right) \qquad i \in I_\ell\,. \tag{11}$$

Here, ℓ is the size of each vehicle. The solution to (11) allows to construct the approximations

$$\rho_\ell(t, x) = \sum_{i \in I_\ell} \frac{\ell}{x_{i+1}(t) - x_i(t)}\, \chi_{[x_i(t), x_{i+1}(t)[}(x) \tag{12}$$

that converge to solutions to (10) as $\ell \to 0$. The results in [2] allow to comprehend in the Follow-the-Leader model (11) the case of I_ℓ being infinite, i.e., $I_\ell = \mathbb{N}$ or also $I_\ell = \mathbb{Z}$, so that no driver has the *a priori* privilege to drive at constant maximal speed, i.e., there is no leader. As a consequence, this micro-macro limit can be extended to the general case of (10), without ruling out vacuum and allowing the initial data to have infinite mass. As shown in [1], the micro-macro limit thus results being an effective technique to prove general existence results for (10).

[1] R. M. Colombo and G. Guerra and F. Marcellini. Existence of Solutions through Micro-Macro Limits in General Traffic Models *In preparation*, 2025.

[2] F. Marcellini The Follow-the-Leader Model without a Leader: an Infinite-Dimensional Cauchy Problem *J. Math. Anal. Appl.*, 495, 2021.

[3] R. M. Colombo, F. Marcellini, and M. Rascle. A 2-phase Traffic Model Based on a Speed Bound. *SIAM J. Appl. Math.*, 70(7):2652–2666, 2010.

Nonlocal conservation laws with p-norm, the singular limit problem and applications to traffic flow

Keimer, Alexander

University of Rostock, Germany

Monday, May 25, 16:50-17:15, M17.52 – 5.019

We study scalar nonlocal conservation laws with the p-norm. Here, "nonlocal" means that the velocity of the conservation law depends on an integral term in space. Typically, the nonlocal term consists of integrating the solution in L^1, whereas here we will investigate the case when the solution is integrated in the L^p-norm (including the L^p-metric for $p \in (0,1)$).

We then look into two singular limits: First, the convergence to the corresponding entropy solution when the nonlocal kernel approaches a Dirac distribution, and second, the convergence of solutions (and first their identification) for $p \to 0$.

Travelling waves and well-posedness in periodic nonlocal-local traffic models

Nocita, Claudia

University of Milano-Bicocca, Italy

Monday, May 25, 17:15-17:40, M17.52 – 5.019

Autonomous vehicles are likely to decide their driving strategy on the basis of more information than that available to human drivers. We are thus led to describe traffic through a model consisting of a *"farsighted"* population and a *"myopic"* one. In analytical terms, this leads to the mixed nonlocal-local system of conservation laws

$$\begin{cases} \partial_t \rho + \partial_x \left(\rho \; v\left((\rho + r) * \eta\right)\right) = 0 & \rho \text{ farsighted population density} \\ \partial_t r + \partial_x \left(r \; v(\rho + r)\right) = 0 & r \text{ myopic population density} \end{cases}$$

A first well-posedness result for (a generalized version of) this system is presented in [2]. Global in time existence, uniqueness and continuous dependence on the initial datum are obtained through the interplay of nonlocal and local techniques, largely relying on the stability estimates in [1, 5] and [4].

Various qualitative properties of the solutions, as well as control problems — such as stabilization effects induced by the presence of the farsighted vehicles — are better studied in a periodic setting. To this setting, hardly considered in the current nonlocal literature, well-posedness is extended in [3].

A study of travelling waves for purely nonlocal traffic models was initiated in [6] in the case of anysotropic kernels. In [3], requiring the kernel to be merely smooth and compactly supported, a variety of non trivial travelling wave solutions to the mixed nonlocal-local problem above are proved to be possible as soon as r is positive almost everywhere.

Numerical simulations exemplify this qualitative behaviour and suggest further open problems.

The work is a joint collaboration with Rinaldo M. Colombo (University of Brescia) and Mauro Garavello (University of Milano-Bicocca).

[1] R. M. Colombo, M. Garavello, and C. Nocita. General stability estimates in nonlocal traffic models for several populations. *Nonlinear Differ. Equ. Appl.*, 32(2):29, 2025.

[2] R. M. Colombo, M. Garavello, and C. Nocita. Coexisting automated and human-driven vehicles: Well-posedness of a mixed nonlocal-local traffic model. *J. Math. Anal. Appl.*, 557(1):Paper No. 130263, 2026.

[3] R. M. Colombo, M. Garavello, and C. Nocita. Travelling wave profiles for a mixed nonlocal-local traffic model. *In progress*, 2026.

[4] R. M. Colombo, M. Mercier, and M. D. Rosini. Stability and total variation estimates on general scalar balance laws. *Commun. Math. Sci.*, 7(1):37–65, 2009.

[5] A. Keimer and L. Pflug. Existence, uniqueness and regularity results on nonlocal balance laws. *J. Differ. Equations*, 263(7):4023–4069, 2017.

[6] J. Ridder and W. Shen. Traveling waves for nonlocal models of traffic flow. *Discrete Contin. Dyn. Syst.*, 39(7):4001–4040, 2019.

Conservation laws with coinciding smooth solutions in fluid dynamics and traffic flow

Guerra, Graziano; Rinaldo M., Colombo; Francesca, Marcellini

University of Milano-Bicocca, Italy

Monday, May 25, 17:40-18:05, M17.52 – 5.019

We consider two one-dimensional hyperbolic systems of conservation laws sharing the same eigenvalues and right eigenvectors, but possibly differing in their shock curves, and hence in their conserved quantities. We show that, for the same initial data, their solutions differ only at third order in the total variation of the initial datum.

This result leads to improved estimates on the distance between solutions of related models, including the isentropic and non-isentropic inviscid Euler equations under general equations of state, as well as Di Perna's approximation of the p-system.

A large class of macroscopic traffic flow models involves a "Lagrangian marker" representing the driving style of drivers. These models admit different conservative formulations that share the same eigenvalues and right eigenvectors. We estimate the distance between the corresponding solutions and show that there exists a unique conservative form in which the Lagrangian marker preserves its physical meaning also in the presence of discontinuities.

[1] R. M. Colombo and G. Guerra and F. Marcellini. Existence of Solutions through Micro-Macro Limits in General Traffic Models *In preparation*, 2025.

[2] R. M. Colombo and G. Guerra. Conservation Laws with Coinciding Smooth Solutions but Different Conserved Variables *Zeitschrift für Angewandte Mathematik und Physik*, 69(2):47, 2018.

P9-1 Higher Order Space Discretizations

An entropy stable discontinuous Galerkin spectral element method on heterogeneous grids

Keim, Jens; Schwarz, Anna; Kopper, Patrick; Rohde, Christian; Beck, Andrea

University of Stuttgart, Germany

Monday, May 25, 14:00-14:25, M17.92 – 9.019

The discontinuous Galerkin spectral element method (DGSEM) has proven to be a highly efficient numerical scheme for the approximation of smooth multiscale problems, e.g., turbulence in the present case. Key aspects for the high efficiency are its excellent parallelization properties and the use of a tensor-product ansatz, where the multidimensional space operator is decomposed into a sequence of one-dimensional operations. Efficient and robust application-oriented large eddy simulations (LES) of turbulence were enabled by the use of entropy-stable DGSEM in a flux differencing formulation. However, for the discretization of complex geometries, more flexible element types, e.g., simplexes, are in general more favorable. In a recent work, Montoya and Zingg [1] presented an efficient entropy stable flux differencing DGSEM formulation on triangular and tetrahedral elements. To benefit from the computational efficiency of the tensor-product ansatz, the authors used a collapsed coordinate transformation on Legendre–Gauss-Lobatto and Legendre-Gauss-Radau nodes. The authors were able to show that their scheme is computationally more efficient as previously developed entropy stable schemes which utilize a full-order basis. Moreover, the intrinsic drawback of small time steps in the collapsed coordinates formulation was resolved by the temporal evolution of modal degrees of freedom (DOFs) instead of nodal ones.

In practical applications such as the flow around an airfoil in the majority of the computational domain, the use of, e.g., hexahedral elements can be favorable and only in a small subset, e.g., simplexes are required. For this, the approach in [1] is extended to triangular prism, pyramid and tetrahedral elements on Legendre–Gauss nodes to enable an efficient entropy stable DGSEM on heterogeneous grids. To avoid any load imbalances due to the additional modal switching of the residual on non-hexahedral elements, a time measurements based partitioning technique is employed for the load balancing.

In the present talk, an entropy stable DGSE scheme on heterogeneous meshes is presented [1]. A particular focus is placed on the collapsed coordinates transformation, the interelement flux computation between different elements types, the modal time-update of the non-hexahedral elements, and the load balancing strategy. The free-stream preservation and the convergence properties are demonstrated based on numerical experiments and the excellent scaling properties of the DGSEM are shown. This talk concludes with several application-oriented numerical problems.

[1] T. Montoya and D. W. Zingg. Efficient entropy-stable discontinuous spectral-element methods using tensor-product summation-by-parts operators on triangles and tetrahedra. *Journal of Computational Physics*, 516:113360, 2024.

[2] J. Keim, A. Schwarz, P. Kopper, M. Blind, C. Rohde, A. Beck. Entropy stable high-order discontinuous Galerkin spectral-element methods on curvilinear, hybrid meshes, arxiv:2507.04334, 2025.

Beyond polynomials: SBP operators for general function spaces

Glaubitz, Jan

Linköping University, Sweden

Monday, May 25, 14:25-14:50, M17.92 – 9.019

Solving time-dependent partial differential equations (PDEs) is vital in many fields. Preserving the critical structures of PDEs during discretization enhances the accuracy and robustness of numerical methods, particularly for hyperbolic and advection-dominated problems. Summation-by-parts (SBP) operators, which discretely mimic integration by parts, play a crucial role in this context. They facilitate the transfer of stability estimates from the continuous to the discrete level. While traditional SBP operators assume polynomial approximations of PDE solutions, there's an increasing demand for non-polynomial methods. Radial basis function (RBF) schemes and neural network-based approximations are notable examples. In this talk, I will develop a theory of function-space SBP (FSBP) operators [1, 2, 3, 4, 5] that extend the structure-preserving properties of polynomial SBP operators to a broader class of function spaces, thereby increasing flexibility and applicability.

This talk is based on joint work with Armin Iske, Joshua Lampert, Jan Nordström, and Philipp Öffner.

[1] J. Glaubitz, J. Nordström, and P. Öffner. Summation-by-parts operators for general function spaces. *SIAM Journal on Numerical Analysis*, 61(2):733–754, 2023.

[2] J. Glaubitz, S.-C. Klein, J. Nordström, and P. Öffner. Multi-dimensional summation-by-parts operators for general function spaces: Theory and construction. *Journal of Computational Physics*, 491:112370, 2023.

[3] J. Glaubitz, J. Nordström, and P. Öffner. Energy-stable global radial basis function methods on summation-by-parts form. *Journal of Scientific Computing*, 98(1):30, 2024.

[4] J. Glaubitz, S.-C. Klein, J. Nordström, and P. Öffner. Summation-by-parts operators for general function spaces: The second derivative. *Journal of Computational Physics*, 504:112889, 2024.

[5] J. Glaubitz, J. Nordström, and P. Öffner. An optimization-based construction procedure for function space-based summation-by-parts operators on arbitrary grids. *Journal of Scientific Computing*, 105(3):83, 2025.

SIAC filtering: Recasting mesh nonuniformities as smooth transformations

Stuhlmacher, Roman; Ryan, Jennifer

KTH Royal Institute of Technology, Sweden

Monday, May 25, 14:50-15:15, M17.92 – 9.019

Smoothness Increasing Accuracy Conserving (SIAC) filters are convolution kernels which are traditionally used as a post-processing tool to reduce the error, increase the smoothness and enhance the convergence rate of finite element approximations of Partial Differential Equations (PDE).

They have been extensively analyzed in the setting of discontinuous Galerkin methods for hyperbolic conservation laws. In this context, their application can improve the order of accuracy on translation-invariant meshes from $p+1$ to $2p+1$ for polynomials of degree p. This even holds for nonlinear hyperbolic problems, in regions where the solution remains smooth.

However, the performance of the post-processing and its ability to recover superconvergence is strongly dependent on the mesh uniformity. While existing strategies for SIAC filtering on nonuniform meshes can lead to superconvergence of order $2p+1$ on smoothly varying meshes, the rate of superconvergence is a priori unknown and might drop to $p+1$, on randomly perturbed structured meshes.

We propose a novel strategy to apply SIAC filtering on nonuniform but structured meshes. The approach is based on the idea of transforming the structured mesh to a translation invariant one, where a SIAC filter is applied. The transformation is is required to be a smooth diffeomorphism to ensure the rate improvement. We provide a theoretical justification for the approach, discuss the assumptions and provide a method to construct the transformation.

The transformation eliminates the need to identify integration regions for individual physical integration points, thereby simplifying the pre-processing work and enabling uniform reuse of local post-processing operations in the interior and the boundary. We provide numerical examples to demonstrate the performance of the approach, suggesting that by using an appropriate transformation, superconvergence of order $2p+1$ can be recovered.

Beyond polynomials: entropy stable and conservative discontinuous Galerkin methods with non-polynomial function spaces

Haase, Henry Morten

Linköping University, Sweden

Monday, May 25, 15:15-15:40, M17.92 – 9.019

We develop entropy stable and conservative discontinuous Galerkin methods based on non-polynomial approximation spaces. Specifically, we consider adaptive approximation spaces that vary spatially (from element to element) and temporally (from time step to time step) to better match the underlying solution. Norm-diagonal function space summation-by-parts (FSBP) operators and flux differencing are the building blocks ensuring conservation and entropy stability. FSBP operators are a generalization of polynomial summation-by-parts (SBP) operators admitting non-polynomial approximation spaces. It has been shown that under mild conditions the conservation properties of SBP operators also apply to FSBP operators. In a next step, we transfer known flux differencing methods for SBP operators to FSBP operators. Numerical simulations are presented to emphasize advantages of the technique proposed here.

P9-2 Higher Order Space Discretizations

Entropy-stable high-order shock capturing on heterogeneous grids for discontinuous Galerkin spectral element methods

Schwarz, Anna; Keim, Jens; Beck, Andrea

University of Stuttgart, Germany

Monday, May 25, 16:00-16:25, M17.92 – 9.019

The discontinuous Galerkin method allows a straightforward construction of efficient high-order schemes on arbitrary grids for hyperbolic partial differential equations (PDEs) such as the Euler equations. However, hyperbolic PDEs admit discontinuities in the solution, even if the initial solution is smooth. Moreover, it is well known that high-order operators induce oscillations at discontinuities, called Gibbs phenomena. Hence, adequate numerical treatment is necessary to detect and handle discontinuities. One example is a switching procedure where a troubled DG cell is switched to a low-order finite volume scheme, where the latter is defined on an equidistant point set. This approach enables a highly accurate and localized shock capturing. However, implementations of h-adaptive FV subcell schemes for DGSEM and related DG schemes were so far limited to hexahedral and tetrahedral elements.

In this talk, a finite volume subcell shock capturing procedure for the entropy-stable discontinuous Galerkin spectral element method on heterogeneous grids, building on the work presented in [1], is proposed. The proposed scheme is based on an h-adaptive entropy-stable second-order total variational diminishing finite volume operator defined on an equidistant point set. The capabilities of the shock capturing procedure are demonstrated on a series of well-known numerical experiments including an application-oriented numerical problem.

[1] J. Keim, A. Schwarz, P. Kopper et al. Entropy stable high-order discontinuous Galerkin spectral-element methods on curvilinear, hybrid meshes, arxiv:2507.04334, 2025.

Nonlinearly stable open boundary conditions for shallow water flows

Winters, Andrew

Linköping University, Sweden

Monday, May 25, 16:25-16:50, M17.92 – 9.019

Provably stable open boundary treatments for nonlinear initial–boundary value problems are both essential and challenging. Recent work has introduced a systematic framework based on the nonlinear energy method for constructing provably stable simultaneous approximation terms (SATs) that weakly impose boundary conditions for a broad class of nonlinear partial differential equations.

For the shallow water equations, non-constant bottom topography $b(x, y)$ alters total energy conservation and stability through an additional potential energy contribution. This modification influences the skew-symmetric formulation as well as the associated (nonlinear) characteristic variables. We show how these changes affect the construction of nonlinearly stable SATs and demonstrate how the nonlinear energy method can be interpreted within a standard mathematical entropy framework. This interpretation is developed in the setting of the two-dimensional shallow water equations in curvilinear coordinates.

In the second part of the talk, we present a reformulation of provably stable open boundary SATs as equivalent numerical fluxes that depend on both external boundary data and interior

solution values. The resulting flux formulation is readily compatible with a range of discretization techniques, including finite volume and discontinuous Galerkin spectral element methods. Incorporating the proposed stable boundary treatment into existing codes requires only the implementation of a new numerical flux routine.

Finally, we provide numerical experiments demonstrating the stability, accuracy, and practical performance of the proposed boundary treatments in high-order discontinuous Galerkin spectral element simulations.

The high order unfitted discontinuous Galerkin methods for hyperbolic conservation laws in high dimension

Fu, Pei

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Monday, May 25, 16:50-17:15, M17.92 – 9.019

This talk will present a family of high order cut finite element methods based on the discontinuous Galerkin (DG) framework for hyperbolic problems in high dimensions, including scalar problems and system problems. To avoid the small-cut-cell issue, a ghost penalty stabilization is employed to stabilize the scheme. The strong stability-preserving Runge–Kutta method is used for time discretization, in which the time step is chosen independently of the size of the cut elements. We show that the proposed methods possess stability and accuracy properties as the standard DG methods on fitted meshes. We also prove and verify that the scheme preserves the bound-preserving property for scalar conservation laws when a flux limiting work is applied. For systems problems, the strategy to control the numerical oscillations and capture the shock will be shown. Finally, numerical experiments will be given to demonstrate that the proposed methods achieve high-order accuracy for smooth solutions and perform well for problems with discontinuities. This work is collaborated with Gunilla Kreiss, Sara Zahedi and Zelin Xin.

Stabilized P2-DG method with artificial viscosity for steady hyperbolic conservation laws

Liu, Tiegang

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Monday, May 25, 17:15-17:40, M17.92 – 9.019

It is well-known that convergence to steady state is notoriously hard when a high order discontinuous Galerkin (DG) method is applied for transonic and supersonic flows in polynomial space even with post-processing such as limiters and positivity preservation. The DG method discretizes the hyperbolic conservation laws in space in advance to obtain a system of first-order ordinary differential equations in time. As a result, the steady-state solution of the DG method is equivalent to the equilibrium point of this system. In this work, we analyze the stability of DG methods in the view point of dynamical systems for the scalar conservation law. We show that the steady-state solution of the 3rd-order DG method is not always stable in the presence of shock waves, and then we propose an artificial viscosity to stabilize the DG method and show that the artificial viscosity has to be order one of the mesh size to improve stability. To maintain higher order accuracy, the proposed artificial viscosity is only applied in the vicinities of shock waves together with a shock-wave indicator. Numerical results are given to verify theoretical analysis.

Several transonic/supersonic flow test cases are also present to demonstrate the effectiveness of the present artificial viscosity.

A dispersive generalized Riemann problem for Galerkin methods

Tirapelle, Michel; Keim, Jens; Rohde, Christian; Beck, Andrea
University of Stuttgart, Germany

Monday, May 25, 17:40-18:05, M17.92 – 9.019

Fluxes based on the solution of the generalized Riemann problem (GRP) play a central role in the numerical approximation of hyperbolic and diffusive partial differential equations, where they provide a systematic framework for constructing high-order, locally accurate interface fluxes [1], [2]. Despite their success in these settings, their extension to dispersive systems, characterized by higher-order spatial derivatives and oscillatory wave propagation, which are essential in physics for describing quantum mechanics, nonlinear optics, and water waves, and for providing key insight into soliton dynamics, scattering, and long-time stability, has not been thoroughly investigated. The fundamental challenge lies in the fact that dispersive terms alter the local wave structure and introduce additional analytical and numerical difficulties that are not present in purely hyperbolic or diffusive problems.

In the present talk, we introduce the dispersive generalized Riemann problem (disGRP), a novel framework designed to construct high-order numerical fluxes for dispersive equations. The proposed approach is developed within a discontinuous Galerkin (DG) formulation that relies on repeated integration by parts to naturally accommodate higher-order spatial derivatives. Importantly, the method avoids the introduction of auxiliary variables and does not require rewriting the original equation as an equivalent first-order system, thereby preserving the compactness and structural properties of the underlying model.

As a simplified, but mathematically well understood model problem, we consider the Korteweg–de Vries (KdV) equation, a prototypical nonlinear dispersive equation that exhibits soliton solutions and dispersive wave phenomena. Within this framework, we derive high-order interface fluxes based on the disGRP formulation and integrate them into the DG scheme. We provide a rigorous stability analysis of the proposed method. In particular, we establish L^2-stability under suitable assumptions on the numerical fluxes and discretization parameters. Additionally, a Neumann (Fourier) stability analysis is carried out to investigate the linearized behavior of the scheme and to characterize its stability region and dispersion properties.

We conclude our talk with a comparison between our solution and the one presented in [3], assessing accuracy, convergence behavior, and long-time performance of the method. These experiments demonstrate the high-order accuracy and strong resolution capabilities of the proposed scheme, while also highlighting its limitations, particularly in regimes involving strong nonlinear interactions or under-resolved dispersive structures. The results confirm that the disGRP framework provides a promising and systematic pathway for extending GRP-based flux constructions to nonlinear dispersive systems.

[1] Gregor Gassner, Frieder Lörcher and Claus-Dieter Munz. A contribution to the construction of diffusion fluxes for finite volume and discontinuous Galerkin schemes. *Journal of Computational Physics*, 224 1049-1063, 2007.

[2] Eleuterio F. Toro. Riemann solvers and numerical methods for fluid dynamics, a practical introduction. Springer, 1999.

[3] Yingda Cheng and Chi-Wang Shu. A discontinuous Galerkin finite element method for time dependent partial differential equations with higher order derivatives,. *Mathematics of Computation*, Volume 77, Number 262, Pages 699-730, 2008.

P9-3 Higher Order Space Discretizations

A structure-preserving compact high-order method for multi-dimensional hyperbolic conservation laws

Klingenberg, Christian

Würzburg University, Germany

Thursday, May 28, 14:00-14:25, M17.92 – 9.019

Multi-dimensional conservation laws possess many more phenomena than their one-dimensional counterparts, for example turbulent flow with vortices and non-trivial stationary states. At the same time, computational resources are limited, and refinement is particularly costly for multi-dimensional simulations.

In this talk we propose a new hybrid finite element - finite volume method that achieves upwinding by locally evolving continuous data, instead of solving a Riemann problem. Its degrees of freedom are cell averages/moments and point values located at cell interfaces, the latter being shared between adjacent cells. This method can be found in the literature under the name of *Active Flux* and has been shown to achieve superior results on coarse grids, because it is structure preserving.

We shall present this numerical method applied to multi-dimensional systems of conservation laws, in particular the multi-dimensional Euler [1] and MHD-equations [2] and we shall also analyze the structure preservation properties of Active Flux [3].

This is joint work among others with *Wasilij Barsukow* (CNRS/University of Bordeaux, France) and *Junming Duan* (Chinese University of Hingkong).

[1] Junming Duan, Wasilij Barsukow, Christian Klingenberg: Active flux methods for hyperbolic conservation laws - flux vector splitting and bound-preservation *SIAM Journal on Scientific Computing*, vol. 47, no. 2, 2025.

[2] Junming Duan, Praveen Chandrashekar; Christian Klingenberg: Active flux for ideal magnetohydrodynamics: A positivity-preserving scheme with the Godunov-Powell source term *arXiv:2506.04857*, 2025.

[3] Wasilij Barsukow, Janina Kern, Christian Klingenberg, Lisa Lechner: Analysis of the multidimensional semi-discrete Active Flux method using the Fourier transform *Communications on Applied Mathematics and Computation*, 2025.

High-order flux splitting schemes for the Euler equations of gas dynamics

Chu, Shaoshuai; Herty, Michael; Toro Eleuterio Francisco

RWTH Aachen University, Germany

Thursday, May 28, 14:25-14:50, M17.92 – 9.019

We develop high-order flux splitting schemes for the one- and two-dimensional Euler equations of gas dynamics. The proposed schemes are high-order extensions of the existing first-order flux splitting schemes introduced in [E. F. Toro, M. E. Vázquez-Cendón, Comput. & Fluids, 70 (2012), pp. 1–12], where the Euler equations of gas dynamics are split into two subsystems: the advection and pressure systems. In this paper, we formulate the TV splitting within the semi-discrete framework to extend it to higher orders of accuracy for the first time. The second-order

extension is obtained by using piecewise linear interpolant to reconstruct the one-sided point values of the unknowns. The third- and fifth-order schemes are developed using the finite-difference alternative weighted essentially non-oscillatory (A-WENO) framework, which is particularly effective in handling multidimensional problems and provides a more straightforward approach to constructing higher-order WENO schemes. These extensions significantly improve the resolution of discontinuities and the accuracy of numerical solutions, as demonstrated by a series of numerical experiments of both the one- and two-dimensional Euler equations of gas dynamics.

An adaptive spectral element method for systems of conservation laws

Colera–Rico, Manuel; Dolejší, Vít

Universidad Politécnica de Madrid, Spain

Thursday, May 28, 14:50-15:15, M17.92 – 9.019

A novel method for the solution of systems of conservation laws is presented [1]. For the space discretization, the scheme considers high-order continuous finite elements stabilized via subgrid modeling, as well as highly anisotropic adaptive meshes in order to capture any sharp features in the solution with a minimum number of mesh elements. Time integration is carried out via linearly implicit formulas, which allow for large time steps and require only the solution of a few linear systems per solved time interval. Numerical experiments, which include a hard case of nonconvex flux and the Euler equations for compressible flows, were performed with up to eighth-degree elements and a third-order time marching formula in order to assess the capabilities of the method.

Acknowledgements. Project TED2021-129446B-C43 from Spain's "Ministerio de Ciencia e Innovación", and "Programa Propio" grant to M. Colera from "Universidad Politécnica de Madrid". V. Dolejší acknowledges the membership in the Nečas Center for Mathematical Modeling.

[1] M. Colera and V. Dolejší. An adaptive spectral element method for systems of conservation laws. *Journal of Computational Physics*, 541:114322, 2025.

A high-order structure-preserving scheme for a model for two-layer thin film flow

Zawallich, Jan

Clausthal University of Technology, Germany

Thursday, May 28, 15:15-15:40, M17.92 – 9.019

In this talk, we study a nonlinear first-order hyperbolic model describing the flow of a two-layer thin film with a perfectly soluble anti-surfactant. We construct a high-order entropy-conservative (and entropy-dissipative) finite element method for this system. The approach is based on Tadmor's framework, where entropy-conservative (and entropy-stable) numerical fluxes are developed and incorporated into a flux-difference formulation within a discontinuous Galerkin setting.

A central aspect of this work is the derivation of the numerical fluxes and the challenges that arise from the fact that not all relevant variables, e.g.entropy, primitive, and conservativ, are available in closed form. We show how these difficulties can be addressed and conclude by presenting numerical results.

P9-4 Higher Order Space Discretizations

Well-balanced high-order finite-difference methods based on a local flux formulation

Parés, Carlos; M. Kazolea, B. Ren

University of Málaga, Spain

Thursday, May 28, 16:00-16:25, M17.92 – 9.019

High-order well-balanced WENO schemes have already been developed for hyperbolic conservation laws (see for example [4]), where the effects of convective fluxes and source terms cancel each other for still water and for moving shallow water(see for example. [5]).

Different strategies have been discussed in the literature to extend high-order WENO finite-difference methods to hyperbolic systems with source terms and/or non-conservative products. In [2], a strategy based on the so-called Local Flux approach was introduced. In this framework, source terms and non-conservative products are represented as the derivative of a generalized flux function that, unlike the Global Flux approach [1], is defined locally based on the choice of a family of paths. Two different techniques were proposed in that work to achieve the well-balanced property. First, the technique developed in [3], based on the computation of local equilibria, was extended. Second, a method relying on an appropriate choice of the family of paths was introduced. This strategy allows non-conservative products to be written as the derivative of a generalized flux function that is defined locally on the basis of the selected family of paths. WENO reconstructions are then applied to this generalized flux.

The goal of this work is to further investigate this approach, which in principle allows the design of well-balanced methods without the additional computational cost associated with computing local equilibria. The approach is tested to different hyperbolic systems like the Burger's equations, the nonlinear shallow water equations and the two layer shallow water equations.

[1] M. Kazolea, C. Parés and M. Ricchiuto, *Approximate well-balanced WENO finite difference schemes using a global-flux quadrature method with multi-step ODE integrator weights*, Computer and Fluids, 2025.

[2] B. Ren, C. Parés, *High-order WENO finite-difference methods for hyperbolic non conservative systems of partial differential equations*, Journal of Computational Physics, 2025.

[3] C. Parés and C. Parés-Pulido, *Well-balanced high-order finite difference methods for systems of balance laws*, Journal of Computational Physics, 425, 2021.

[4] G. Huang, Y. Xing, and T. Xiong, *High order well-balanced asymptotic preserving finite difference WENO schemes for the shallow water equations in all Froude numbers*, Journal of Computational Physics, 463, 2022.

[5] S. Noelle, Y. Xing, and C.-W. Shu, *High-order well-balanced finite volume WENO schemes for shallow water equation with moving water*, Journal of Computational Physics, 226,2007.

Entropy analysis and entropy stable DG methods for the shallow water moment equations

Careaga, Julio; Ersing, Patrick; Koellermeier, Julian; Winters, Andrew R.

University of Groningen, The Netherlands

Thursday, May 28, 16:25-16:50, M17.92 – 9.019

In this talk, we present the derivation of an energy equation, and entropy variables for the so-called shallow water moment equations (SWME) [3, 4], which are an extension of the well-known shallow water model. In addition, we show that the standard Newtonian slip and Newtonian Manning friction terms are entropy dissipative with respect to the developed entropy variables. Based on the nodal discontinuous Galerkin spectral element method [2], we develop a high order entropy preserving scheme to approximate the SWME. For this, we construct a numerical flux following the derivation of the entropy variables in the continuous case, and such that an entropy preserving condition including non-conservative terms is fulfilled. Furthermore, we prove that the proposed scheme is also well-balanced and entropy stable. The numerical scheme is implemented in Julia making use of the library Trixi. Finally, numerical examples show the reliability of the scheme, validating the preservation of entropy and well-balance of the scheme as well as the expected order of convergence. This presentation is based on the pre-print [1].

Acknowledgements. The authors acknowledge the Dutch Research Council (NWO) for their support through the ENW Vidi project HiWAVE with file number VI.Vidi.233.066.

[1] J. Careaga, P. Ersing, J. Koellermeier and A.R. Winters.Entropy analysis and entropy stable DG methods for the shallow water moment equations. *Arxiv pre-print arXiv:2602.06513*, 2026.

[2] G. Gassner, A. Winters, and D. Kopriva. A well balanced and entropy conservative discontinuous Galerkin spectral element method for the shallow water equations. *Applied Mathematics and Computation*, 272, pp. 291–308, 2016.

[3] J. Koellermeier and M. Rominger. Analysis and Numerical Simulation of Hyperbolic Shallow Water Moment Equations. *Communications in Computational Physics*, 28(3), pp. 1038–1084, 2020.

[4] J. Kowalski and M. Torrilhon, Moment approximations and model cascades for shallow flow. *Communications in Computational Physics*, 25(3), pp. 669–702, 2019.

OEDG: Oscillation-eliminating discontinuous Galerkin method for hyperbolic conservation laws

Peng, Manting; Sun, Zheng; Wu, Kailiang

Southern University of Science and Technology, China

Thursday, May 28, 16:50-17:15, M17.92 – 9.019

Suppressing spurious oscillations is crucial for designing reliable high-order numerical schemes for hyperbolic conservation laws, yet it has been a challenge actively investigated over the past several decades. This paper proposes a novel, robust, and efficient oscillation-eliminating discontinuous Galerkin (OEDG) method on general meshes, motivated by the damping technique (see J. Lu, Y. Liu, and C. W. Shu [SIAM J. Numer. Anal. 59 (2021), pp. 1299–1324]). The OEDG method incorporates an oscillation-eliminating (OE) procedure after each Runge–Kutta stage, and it is devised by alternately evolving the conventional semidiscrete discontinuous Galerkin (DG) scheme and a damping equation. A novel damping operator is carefully designed to possess both scale-invariant and evolution-invariant properties. We rigorously prove the optimal error estimates of the fully discrete OEDG method for smooth solutions of linear scalar conservation laws. This might be the first generic fully discrete error estimate for nonlinear DG schemes with an automatic oscillation control mechanism. The OEDG method exhibits many notable advantages. It effectively eliminates spurious oscillations for challenging problems spanning various scales and wave speeds, without necessitating problem-specific parameters for all the tested cases. It also obviates the need for characteristic decomposition in hyperbolic systems. Furthermore, it retains the key properties of the conventional DG method, such as local conservation, optimal convergence rates, and superconvergence. Moreover, the OEDG method maintains stability under the normal Courant–Friedrichs–Lewy (CFL) condition, even in the presence of strong shocks associated with highly stiff damping terms. The OE procedure is non-intrusive, facilitating seamless integration into existing DG codes as an independent module. Its implementation is straightforward and efficient, involving only simple multiplications of modal coefficients by scalars. The OEDG approach provides new insights into the damping mechanism for oscillation control. It reveals the role of the damping operator as a modal filter, establishing close relations between the damping technique and spectral viscosity techniques. Extensive numerical results validate the theoretical analysis and confirm the effectiveness and advantages of the OEDG method.

Oscillation-eliminating central DG schemes on Cartesian meshes for hyperbolic conservation laws

Caiyou, Yuan

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Thursday, May 28, 17:15-17:40, M17.92 – 9.019

This talk introduces and analyzes the OECDG method, a novel central discontinuous Galerkin (CDG) scheme that integrates the strengths of the oscillation-eliminating (OE) approach [M. Peng, Z. Sun, and K. Wu, *Math. Comp.*, 94 (2025), pp. 1147–1198] within the CDG framework for general hyperbolic conservation laws. The OECDG method incorporates a new OE procedure, along with an innovative dual damping mechanism, to enhance oscillation control in CDG schemes. Unlike OE procedures in standard DG methods that use inter-cell jumps as a modal filter, our new OE mechanism draws inspiration from CDG-based numerical dissipation and employs a convex blending strategy, leveraging overlapping solutions to enhance stencil com-

pactness and resolution without characteristic decomposition. We rigorously derive optimal error estimates for the fully discrete OECDG method through several key theoretical advancements, filling a gap in the error analysis of fully discrete CDG schemes, including original linear CDG methods without oscillation control. First, we establish the approximate skew-symmetry and weak boundedness of the CDG spatial discretization, which underpins the fully discrete stability analysis. Utilizing these properties, we then prove the linear stability of CDG methods coupled with Runge–Kutta time discretization via matrix transfer techniques. These foundational results enable us to derive fully discrete optimal error estimates for the OECDG method—a challenging task due to the method's nonlinear nature, even for linear advection equations. Extensive numerical experiments validate the theoretical findings and demonstrate the efficacy of the OECDG method across a variety of hyperbolic conservation laws, including linear and nonlinear problems such as the convection equation, Burgers equation, a traffic flow model, and Euler equations. The results confirm the OECDG method's ability to achieve optimal convergence rates, robustly eliminate spurious oscillations, and accurately capture complex wave structures across diverse test cases.

P10-1 Active Flux/Central-Upwind

Recent developpement on active flux methodology

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University of Zurich, Switzerland; université de Chambéry, France

Monday, May 25, 14:00-14:25, M17.02

Considering an hyperbolic problem defined on a domain Ω of $\mathbb{R}^d$, $d = 1, 2, 3$ covered by polygons, we are interested in methods that use globally continuous approximation of the data, while being able to handle discontinuities. One possible method use as degrees of freedoms the Gauss-Lobatto points on the boundary or the polygons K, and moments in K of the solution. From this, it is possible to use a globally continuous approximation where

- when the polygon is a triangle, for an approximation of order $k + 1$, one uses an approximation space which contains $\mathbb{P}^k$ and is contained in $\mathbb{P}^{k+1}$,
- when a general polygon is used by using the approximation tools defined for Virtual Finite Elements,

To illustrate, let us take the example of third order of accuracy, i.e. $k = 2$. On each face of the polygon, we have 3 Gauss-Lobatto point (defining a quadratic polynomial on this face), and we need one moment, the average value. The average is updated by

$$|K|\frac{d\overline{\mathbf{u}}_K}{dt} + \int_{\partial K} \mathbf{f}(u)\mathbf{n}\, d\gamma = 0$$

which is doable (without Riemann solver) because we know $\mathbf{u}$ on the boundary, and the point values are updated by

$$\frac{d\mathbf{u}_\sigma}{dt} + \sum_{K,\sigma\in K} \Phi_\sigma^K(u) = 0$$

The residuals are approximations of div $\mathbf{f}(\mathbf{u})$ on the polygon K which are consistant with this operator, in the sense that if the solution were linear, and the flux linear with constant Jacobian, the sum $\sum_{K,\sigma\in K} \Phi_\sigma^K(u)$ would exacly give div $\mathbf{f}(\mathbf{u})$. We have shown that this condition is essentially suffisant, together with the classical stability bounds, to get a Lax-Wendroff theorem.

This kind of schemes is within the family of active flux, initially designed by Roe and his co-worker, and has since then received a lot of attention: one may mention the work of Barsukow, Klingenberg, Heltzel in Europe, Wu, Kurganov and others in China.

It turns out that this kind of scheme has a lot of connection with discontinuous Galerkin schemes. Using a Runge Kutta time discretisation, one can reinterpret the above scheme, for a linear flux with constant Jacobian as, for each Runge Kutta cycle, one step of DG followed by a projection of the discontinuous solution onto the solution space. The projector operator can be a simple arithmetic average (at least in 1D), or a more complex one with upwinding flavor. We also have gone further by showing how to properly describe the implementation of boundary conditions in this kind of scheme.

It is well known that solutions of hyperbolic problems are genericaly discontinuous. This implies that some non linear stabilisation mechanism must be added. We have proposed a very simple algorithm to guaranty that the solution stays in the invariant domain, and where in addition a non linear mechanism is proposed to remove spurious oscillations.

In the talk, we will make a coherent summary of all these findings. All this works can be found on ArXiv.

A finite element view on arbitrarily high-order accurate active flux methods

Barsukow, Wasilij

Imperial College London, United Kindgom

Monday, May 25, 14:25-14:50, M17.02

The Active Flux method evolves averages/moments and, additionally, point values at cell interfaces, which is at once reminiscent of Discontinuous and Continuous Galerkin methods. This talk aims at positioning Active Flux amongst existing families of (stabilized) Finite Element methods.

The point values can be updated either directly using so-called evolution operators, or by discretizing spatial derivatives first and using the method of lines to integrate in time. This latter, semi-discrete approach is the focus of this talk. The point values are shared among adjacent cells, giving rise to a continuous approximation of the solution. The key difference to classical continuous Galerkin methods is that there is no need to invert any global (or even local) mass matrix: Active Flux does not start out from a variational form and, instead, the update of the degrees of freedom is written down directly. At the same time Active Flux is conservative and can be seen as a high-order extension of Finite Volume, while keeping the stencil very compact, similar to Discontinuous Galerkin methods. This talk will present versions of Active Flux of arbitrary order of accuracy and show its relations to Continuous and Discontinuous Galerkin mehods. In particular, it will be shown that Active Flux can be understood as a Petrov-Galerkin method with a biorthogonal family of continuous basis functions and discontinuous test functions.

Comparing the computational complexity of active flux and discontinuous Galerkin methods

Krotsch, Simon; Barsukow, Wasilij; Klingenberg, Christian

University of Würzburg, Germany

Monday, May 25, 14:50-15:15, M17.02

The Active Flux method can be viewed as an extension of finite volume methods. The degrees of freedom of the semi-discrete Active Flux method of arbitrarily high order consist of moments and point values located at the cell interfaces and shared between adjacent cells. This allows for a globally continuous reconstruction. Without modification, the Active Flux method is structure-preserving and asymptotic-preserving for the hyperbolic heat equation.

In this talk, we analyze the computational complexity of Active Flux methods of different orders in two spatial dimensions, both theoretically and through numerical experiments, and compare it to that of tensorial modal Discontinuous Galerkin methods. Both methods are implemented in a unified way, using identical routines for equivalent computational components wherever applicable.

Active flux method for hyperbolic problems using bicharacteristic evolution operators and WENO reconstruction

Tang, Zhuyan; Lukáčová-Medviďová, Mária

Johannes Gutenberg-Universität Mainz, Germany

Monday, May 25, 15:15-15:40, M17.02

We present a third-order fully discrete Active Flux method based on multidimensional approximate evolution operators derived from bicharacteristic theory. The operators evolve edge values treated as independent variables for flux evaluation, which increases the degrees of freedom but leads to restrictive CFL stability constraints. To improve stability, we derived a new evolution operator that exactly reproduces one-dimensional quadratic plane waves and successfully yields smaller global errors. In addition, the impact of various third-order spatial reconstructions, such as the central weighted essentially non-oscillatory (CWENO) technique on this property of the scheme are examined. Furthermore, based on the framework of dissipative measure-valued solutions and the weak–strong uniqueness principle, we show that the first-order Active Flux finite volume method converges to a strong solution of the multidimensional Euler system, provided such a solution exists.

This is a joint work with C. Helzel and A. Porfetye (Düsseldorf) and S. Chu (Aachen). It was supported by the German Science Foundation (DFG) under SPP 2410: Hyperbolic Balance Laws: Complexity, Scales and Randomness, project numbers 525800857 and 525853336.

P10-2 Active Flux/Central-Upwind

An active flux method for compressible Navier-Stokes equations

Duan, Junming; Barsukow, Wasilij; Christian, Klingenberg
The Chinese University of Hong Kong (Shenzhen), P.R. China
Monday, May 25, 16:00-16:25, M17.02

The active flux (AF) method is a compact high-order finite volume method, where cell averages and point values at cell interfaces are evolved simultaneously. In this talk, we will present a high-order AF method for compressible Navier-Stokes equations. The evolution of cell averages follows the standard finite volume method, ensuring conservation, where the fluxes are directly obtained from the point values at the cell interfaces, and Riemann solvers are not required. The point values are updated using suitable finite difference operators. To maintain the original compact stencil of the AF methods, the proposed method is built on compact finite difference operators for both first and second derivatives. Several numerical experiments validate the high-order accuracy and efficacy of the method.

A locally divergence-free local characteristic decomposition based path-conservative central-upwind scheme for ideal magnetohydrodynamics

Na, Mingye; Chu, Shaoshuai; Kurganov, Alexander; Lukáčová-Medviďová, Mária
Johannes Gutenberg University Mainz, Germany
Monday, May 25, 16:25-16:50, M17.02

We introduce a novel, low-dissipation numerical scheme for solving the ideal magnetohydrodynamics (MHD) equations—a hyperbolic system central to modeling astrophysical, space, and plasma phenomena. The proposed method is a low-dissipation extension of the recently developed scheme in [2], in which a novel second-order locally divergence-free PCCU scheme was presented for both ideal and shallow water MHD. To reduce numerical dissipation and enhance the resolution of numerical results, the local characteristic decomposition (LCD) is incorporated into the PCCU framework, aligning computations with the characteristic structure of the system based on both conservative and primitive variables. The scheme is applied to the MHD equations with Godunov-Powell terms, which aid in controlling divergence errors and ensure a complete eigenstructure required for the LCD approach. By augmenting the system with evolution equations for the magnetic field derivatives and employing a specially designed, non-oscillatory piecewise linear reconstruction, the method maintains the local divergence-free property of the magnetic field. A series of benchmark tests demonstrates that the LCD-PCCU scheme achieves higher resolution compared to its predecessor, while effectively preserving positivity and controlling magnetic divergence.

Acknowledgements. This project has been funded by the Gutenberg Research Fellowship of JGU Mainz and TRR 146 Multiscale Simulation Methods for Soft Matter Systems.

[1] A. Chertock, A. Kurganov, M. Redle, and K. Wu. A new locally divergence-free path-conservative central-upwind scheme for ideal and shallow water magnetohydrodynamics. *SIAM Journal on Scientific Computing*, 46 (2024), pp. A1998–A2024.

A low-dissipation central-upwind schemes for ideal MHD

Cheng, Yu-Chen; Chandrashekar, Praveen; Klingenberg, Christian

University of Würzburg, Germany

Monday, May 25, 16:50-17:15, M17.02

We shall present a central scheme for ideal magnetohydrodynamics (MHD) called the low-dissipation central-upwind (LDCU) scheme. It is based on [1], where for the Euler equations the contact wave is sharpened.

In this talk, based on the same concept, we extend their LDCU scheme to the MHD system in one- and two-dimension. In the two-dimensional case, we separate the variables into two groups: hydrodynamic and magnetic. To the hydrodynamic variables, we apply the LDCU structure to derive a semi-discrete scheme, and use the Runge-Kutta method to update each time step. In order to maintain the magnetic field divergence-free, we use the constrained transport method for the magnetic field.

To validate our proposed scheme, we apply the scheme to several numerical experiments. The results show that the LDCU correction term makes the contact discontinuity noticeably sharper. Also, the results achieve experimental second-order accuracy.

[1] A. Kurganov and R. Xin. New low-dissipation central-upwind schemes. *Journal of Scientific Computing*, 96(56), 2023.

[2] S. Chu and A. Kurganov and R. Xin. New Low-Dissipation Central-Upwind Schemes. Part II. *Journal of Scientific Computing*, 103(33), 2025.

[3] Y.-C. Cheng and P. Chandrashekar and C. Klingenberg. A low-dissipation central-upwind schemes for ideal MHD, arXiv:[math.NA], 2026.

Positivity-preserving new low-dissipation central-upwind schemes for compressible Euler equations

Xin, Ruixiao; Cui, Shumo; Gu, Yaguang; Kurganov, Alexander; Wu, Kailiang

Southern University of Science and Technology, China

Monday, May 25, 17:15-17:40, M17.02

One major challenge in developing accurate and robust numerical schemes for compressible Euler equations arises due to the potential emergence of discontinuous structures in the solution. Recently proposed low-dissipation central-upwind (LDCU) schemes achieve sharp resolution of these structures without introducing spurious oscillations. However, unlike many other Godunov-type methods, the LDCU schemes cannot be written as a convex combination of first-order positivity-preserving (PP) schemes. Therefore, the PP property of the LDCU schemes cannot be analyzed by standard techniques. In this talk, we overcome this difficulty by first decomposing the studied schemes into a convex combination of several intermediate solution states, and then analyzing their PP properties. The performed analysis helps us to construct PPLDCU schemes for Euler equations of compressible gas dynamics, guaranteeing the positivity of computed density and pressure. To achieve the PP property, the built-in anti-diffusion terms in the two-dimensional case and the piecewise linear reconstruction procedure in both the one- and two-dimensional cases are redesigned. The effectiveness and robustness of the proposed PPLDCU schemes are demonstrated in several challenging numerical examples.

Bound- and positivity-preserving path-conservative central-upwind AWENO scheme for the five-equation model of compressible two-component flows

Fu, Qingcheng; Gu, Yaguang; Kurganov, Alexander; Wang, Bao-Shan

Southern University of Science and Technology, China

Monday, May 25, 17:40-18:05, M17.02

We propose a fifth-order finite-difference bound- and positivity-preserving (BP-P) pathconservative central-upwind alternativeweighted essentially non-oscillatory(PCCU-AWENO) scheme for the five-equation model of compressible two-component flows. The proposed scheme is capable of maintaining the equilibrium near moving material interfaces, ensuring that velocity and pressure remain continuous at these interfaces. In addition, the scheme is designed to preserve the bound of volume fraction and positivity of density and internal energy: these three properties are absolutely essential for robustness of the proposed scheme. To this end, we first prove the BP-P property of the first-order PCCU scheme under a sufficient CFL condition and then introduce high-order BP-P limiters. The performance of the introduced PCCU-AWENO scheme is demonstrated on several one- and two-dimensional challenging numerical examples.

P11-1 Higher Order Time Discretizations

Quinpi, or going implicit for nonlinear hyperbolic equations

Semplice, Matteo; Visconti, Giuseppe; Žeravý, Michal; Puppo, Gabriella

Università dell'Insubria, Como

Thursday, May 28, 14:00-14:25, M17.12 – 1.005

Many interesting applications of hyperbolic systems of equations are stiff, in the sense that restrictive CFL conditions are imposed by fields that one is not really interested in tracking accurately. A typical solution in these cases is to resort to implicit time integration, but in the field of high order accurate numerical schemes for hyperbolic equations this is made very difficult by the extreme nonlinearity of the reconstruction operators.

In this talk I will illustrate an approach, that we called Quinpi, to treat, in the context of finite volume schemes, nonlinear hyperbolic equations with high order accurate implicit timestepping. It is based on a third order Central WENO reconstruction, a third order DIRK time integrator and a novel idea of time limiting. The scheme is linearized as much as possible using first order accurate predictors for Runge-Kutta stages and the final scheme contains only the nonlinearity of the flux function: in particular it can be applied by solving only linear systems for linear equations. Limiting in time is needed to control spurious oscillations arising from the fact that waves can cross more than one computational cell in each timestep: different approaches on this will be discussed.

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Extending Quinpi in 2D: an implicit scheme for nonlinear conservation laws

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Università degli Studi dell'Insubria, Italy

Thursday, May 28, 14:25-14:50, M17.12 – 1.005

The Quinpi approach has been proposed in (Puppo et al., *Comm. Comput. Phys.*, 2024) to integrate systems of stiff conservation laws in one dimension. It is a general high-order implicit method that addresses the problem of integrating a nonlinear equation by introducing a first-order implicit predictor, which is employed in both space and time limiting. In this talk, I will present the multi-dimensional extension. Similarly to the one-dimensional case, the scheme combines a third-order Central WENO-Z reconstruction in space with a third-order Diagonally Implicit Runge-Kutta (DIRK) method for time integration, and a low order predictor to ease the computation of the Runge-Kutta stages. Even applying space-limiting, spurious oscillations may still appear in implicit integration, especially for large time steps. For this reason, a time-limiting procedure inspired by the MOOD technique and based on numerical entropy production together with a cascade of DIRK schemes of decreasing order is applied. The scheme is tested on the Euler equations of gasdynamics, also in low Mach regimes. The numerical tests are performed on both structured and unstructured meshes.

[1] G. Puppo, M. Semplice, and G. Visconti. Quinpi: Integrating Stiff Hyperbolic Systems with Implicit High Order Finite Volume Schemes. *Comm. Computat. Phys.*, 36(1):30–70, 2024.

Stability analysis of ADER-DG methods on classical and degenerate spacetime geometries

Bonafini, Mauro

University of Verona, Italy

Thursday, May 28, 14:50-15:15, M17.12 – 1.005

In this talk, we present a rigorous von Neumann stability analysis of explicit and implicit ADER-DG methods on both classical and degenerate spacetime geometries. We first derive CFL-type stability conditions for the explicit ADER-DG method on classical spacetime elements, confirming bounds widely adopted in the literature while specifying their limitations.

We then extend the analysis to spacetime geometries containing degenerate elements with zero size at the beginning and the end of the time step, but non zero spacetime volume. Such elements arise in direct Arbitrary-Lagrangian-Eulerian (ALE) methods to connect via spacetime control volumes regenerated Voronoi tessellations after a topology change. To isolate their effect on stability, we mimic this setting in one spatial dimension by inserting fictitious degenerate elements between neighboring cells.

Our results show that the presence of degenerate spacetime elements does not introduce additional stability restrictions: the CFL constraints for both explicit and implicit ADER-DG methods coincide with those obtained on classical spacetime geometries. These findings provide a solid theoretical basis for the use of ADER-DG schemes in ALE frameworks involving mesh regeneration and spacetime degeneracies.

This is joint work with E. Gaburro (Verona) and D. Torlo (Rome).

[1] M. Bonafini, D. Torlo, E. Gaburro. *Stability analysis of Arbitrary-Lagrangian-Eulerian ADER-DG methods on classical and degenerate spacetime geometries.* arXiv:2602.09198.

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Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency.

ADER schemes go faster and faster

Chiocchetti, Simone; Gassner, Gregor

University of Cologne, Germany

Thursday, May 28, 15:15-15:40, M17.12 – 1.005

The higher efficiency of ADER numerical schemes [1] with respect to similar semi-discrete methods is often cited as one of their main benefits, and attributed to their communication-avoiding design that makes them better suited for parallelism. Since 2006-2008 [2, 3], thanks to the introduction of a local DG predictor step replacing the Cauchy–Kovalewskaya procedure, these numerical methods offer greater versatility, making them well suited for applications to general PDE systems. This talk focuses on the algorithmic performance of the schemes on the node-level and simplifying their implementation.

In this work, we discuss how numerical methods of the ADER family can be pushed to the extreme for applications in fluid flow requiring high performance. We show how some appropriate choices of basis functions can be used to construct very simple and efficient space-time evolution operators that are valid on arbitrary geometries. We show how partial orthogonalization of the

basis functions and sparsity patterns in the space-time operators, initially studied in this work as a purely implementation-level optimization, can explicitly guide the construction of the scheme, giving rise to a family of methods from which the implementer can choose the "preferable" option depending on hardware-level considerations like minimizing storage requirements or minimizing floating point operations in the DG predictor step which constitutes the core of the scheme. Some basis functions are simpler to evaluate but lead to heavy, dense space-time evolution operators, while appropriately chosen orthogonal basis functions can lead to simpler evolution operators, but require additional storage, memory bandwidth and even compute for the simple evaluation. The talk tries to give some insights on the co-design process of these numerical methods where implementation takes a central role.

Acknowledgements. The Authors acknowledge financial support from the European Union under Marie Skłodowska-Curie Postdoctoral Fellowship (European Union's Horizon Europe Research and Innovation Programme, grant agreement No. 101109532) and from the Deutsche Forschungsgemeinschaft under the excellence cluster EXC 3037, Unser dynamisches Universum (Dynaverse).

[1] V. A. Titarev and E. F. Toro. ADER: Arbitrary High Order Godunov Approach. *Journal of Scientific Computing*, 17(1–4):609–618, 2002.

[2] M. Dumbser and C.-D. Munz. Building Blocks for Arbitrary High Order Discontinuous Galerkin Schemes. *Journal of Scientific Computing*, 27(1–3):215–230, 2006.

[3] M. Dumbser, D. S. Balsara, E. F. Toro, and C.-D. Munz. A unified framework for the construction of one-step finite volume and discontinuous Galerkin schemes on unstructured meshes. *Journal of Computational Physics*, 227(18):8209–8253, 2008.

P12-1 Nonlocal Models

A nonlocal transport model for human blood cell dynamics

Friedrich, Jan; Shi, Wenhui; Stiehl, Thomas

Technical University of Munich, Germany

Monday, May 25, 16:00-16:25, M17.81 – 8.019

Blood-forming stem cells play a key role in diseases such as blood cancers and bone marrow failure syndromes. Mathematical and computational models are versatile tools to study human blood cell dynamics, which cannot be directly observed *in vivo*. Transport partial differential equations can be used to account for recent experimental observations suggesting that progenitor cell maturation is a continuous process in which cells undergo gradual changes in their properties. In this talk, we extend a model of blood cell formation from the literature to account for a biologically motivated regulatory signal that depends on mature and immature blood cells. The transport velocity depends on the maturity level of the cells and on the feedback signal. This results in a hyperbolic balance law with a nonlocal transport term coupled to ordinary differential equations at the boundaries. We will discuss the well-posedness of the model, as well as the stability of the model around suitable stationary states. Furthermore, we present numerical results that accompany the analytical ones.

Acknowledgements. J. F. is supported by the German Research Foundation (DFG) through SPP 2410 'CoScaRa' under grant FR 4850/1-1. J. F., W. S. and T. S. thank the Open Seed Fund 2025 of the RWTH Aachen University for their support under the project grant OPSF884.

A nonlocal toy model for quasigeostrophic thermal active scalar

Abreu, Eduardo; Pérez, John; Quispe-Cuba, Nelson; Valencia-Guevara, Julio

University of Campinas, Brazil

Monday, May 25, 16:25-16:50, M17.81 – 8.019

In this work, we consider the following *partially* nonlocal conservation law posed in $\mathbb{R}^2$,

$$\begin{cases} \partial_t \theta + \nabla \cdot (\theta \boldsymbol{v}) = 0 \\ \boldsymbol{v} = (\Lambda_x^{\alpha-1}\mathcal{H}_x\theta, \Lambda_y^{\alpha-1}\mathcal{H}_y\theta) \\ \theta(x,y,0) = \theta_0(x,y), \end{cases} \tag{13}$$

where $\Lambda_x^{\alpha-1}$, $\mathcal{H}_x$, $\Lambda_y^{\alpha-1}$, and $\mathcal{H}_y$ denote the Riesz potential and the Hilbert transform acting on a single variable, as defined below:

$$\Lambda_x^{\alpha-1}\theta(x,y) = c_\alpha \int_{\mathbb{R}} \frac{\theta(s,y)}{|s-x|^\alpha}\, ds, \quad c_\alpha = \frac{\Gamma\left(\frac{\alpha}{2}\right)}{2^{1-\alpha}\sqrt{\pi}\,\Gamma\left(\frac{1-\alpha}{2}\right)}, \quad \mathcal{H}_x\theta(x,y) = \frac{1}{\pi}\,\text{p. v.} \int_{\mathbb{R}} \frac{\theta(s,y)}{x-s}\, ds. \tag{14}$$

The operators $\Lambda_y^{\alpha-1}$ and $\mathcal{H}_y$ are defined analogously. Roughly speaking, the model (13) can be viewed as a *toy model* for the *2-D quasigeostrophic thermal active scalar system* as introduced in the seminal work of Constantin, Majda, and Esteban (1994) [4] for a striking mathematical study of the interplay between hot and cold air masses, i.e., the apparition in finite time of discontinuous temperature - called Frontogenesis. This work is a first step to establish a well-posedness theory for Model (13) within the framework of Besov spaces $B^{\gamma}_{p,q}$ as discussed recently in [5]. In this talk, we will discuss our recent theoretical advances and numerical methods for the problem. We derive suitable Bernstein-type inequalities and commutator estimates, which

is motivated in a similar fashion as in [2]. Furthermore, we develop a Lagrangian–Eulerian numerical scheme to numerically solve model (13), based on the concept of no-flow curves [1] and an efficient approximation of the nonlocal operators $\Lambda_x^{\alpha-1}$ and $\mathcal{H}_x$, among others (see, e.g. [3]).

[1] Eduardo Abreu, Jorge Agudelo, Wanderson Lambert, and John Perez. A lagrangian–eulerian method on regular triangular grids for hyperbolic problems: Error estimates for the scalar case and a positive principle for multidimensional systems. *Journal of Dynamics and Differential Equations*, 37(1):749–814, 2025.

[2] Eduardo Abreu, John Pérez, Nelson Quispe-Cuba, and Julio Valencia-Guevara. Well-posedness on a large besov-weak-morrey space and novel numerical insights for the inviscid sqg equation. *Submitted*, 2025.

[3] Eduardo Abreu, Julio Valencia-Guevara, Magdalena Huacasi-Machaca, and John Pérez. A numerical scheme for doubly nonlocal conservation laws. *Calcolo*, 61(4):72, 2024.

[4] Peter Constantin, Andrew J Majda, and Esteban Tabak. Formation of strong fronts in the 2-d quasigeostrophic thermal active scalar. *Nonlinearity*, 7(6):1495, 1994.

[5] Lucas CF Ferreira and Jhean E Pérez-López. On the well-posedness of the incompressible euler equations in a larger space of besov–morrey type. *Dynamics of Partial Differential Equations*, 19(1):23–49, 2022.

Encryption and decryption through general non local balance laws

Salvadori, Andrea; Amadori, Debora; Colombo, Rinaldo M.

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Monday, May 25, 16:50-17:15, M17.81 – 8.019

A system of non local balance laws such as

$$\partial_t \rho_i + \operatorname{div}_x \left(\rho_i \, V_i(t, x, \rho * \eta)\right) = F_i(t, x, \rho)$$

generates a process

$$\begin{array}{rccccc} \mathcal{P}\colon & \mathbb{R}^2 & \times (\mathbf{L^1} \cap \mathbf{L}^\infty \cap \mathbf{BV})(\mathbb{R}^n; \mathbb{R}^m) & \to & (\mathbf{L^1} \cap \mathbf{L}^\infty \cap \mathbf{BV})(\mathbb{R}^n; \mathbb{R}^m) \\ & (t_o, t) & \rho_o & \mapsto & \mathcal{P}_{t_o,t}\rho_o \end{array}$$

in the sense that the map $t \mapsto \mathcal{P}_{t_o,t}\rho_o$ is the solution to the balance law above at time t, with initial datum ρ_o assigned at time t_o. This process is *reversible*, hence it is effective as a cryptography tool.

Indeed, a general message can be represented through $\rho_o \in (\mathbf{L^1} \cap \mathbf{L}^\infty \cap \mathbf{BV})(\mathbb{R}^n; \mathbb{R}^m)$: for instance, a text fits in the case $n = 1$, $m = 1$; a stereo sound in $n = 1$, $m \geq 2$; an RGB image in $n = 2$, $m = 3$.

In this ciphering setting, the role of the private key is played by V, η, F and (t_o, t). The sender encodes the message ρ_o by means of $\mathcal{P}$, so that $\overline{\rho} = \mathcal{P}_{t_o,t}\rho_o$ is the encrypted message to be sent. The receiver, aware of V, η, F and (t_o, t), is able to decrypt $\overline{\rho}$ computing $\widetilde{\rho} = \mathcal{P}_{t,t_o}\overline{\rho}$ and reversibility ensures that $\widetilde{\rho} = \rho_o$.

This talk presents analytical results that ground the above ciphering procedure and numerical results that show some of its possible uses. Further qualitative properties are obtained, e.g., a

proposal to *quantitatively* evaluate the quality of this encryption tool. Besides, new problems arise from this non standard perspective, for instance: is the map $\big(V, \eta, F, (t_o, t)\big) \mapsto \overline{\rho}$ surjective for a wide class of ρ_o? A positive answer to this controllability problem would ensure that the present encryption procedure is *perfect*.

The implementation of $\mathcal{P}$ unavoidably leads to numerical errors, so that $\widetilde{\rho}$ is only approximately equal to ρ_o. Can $\|\widetilde{\rho} - \rho_o\|_{\mathbf{L^1}}$ be minimized through the development of *ad hoc* (reversible?) numerical algorithms? What is the computational cost of breaking this cipher? Is it *quantum resistant*?

[1] D. Amadori, R. M. Colombo and A. Salvadori. Non local balance laws in cryptography. *Submitted*, 2025.

[2] R. M. Colombo and A. Salvadori. Non conservative and non local dynamics of clusters. *Submitted*, 2025.

Nonlocal regularizations of the Burgers equation

Viorel, Adrian

Babeş-Bolyai University, Romania

Monday, May 25, 17:15-17:40, M17.81 – 8.019

Nonlocal models have attracted increased interest both from a modeling perspective and because they have been proven [1] to converge to standard local models (like the Burgers equation) when the averaging range shrinks to zero. In this context, the present contribution deals with several nonlocal Burgers models among which

$$\begin{cases} u_t + p_x = 0, \\ \qquad \tau p_t = \varepsilon^2 p_{xx} + \left(\frac{1}{2}u^2 - p\right). \end{cases} \tag{15}$$

and

$$\begin{cases} u_t + p u_x = 0, \\ \qquad \tau p_t = \varepsilon^2 p_{xx} + (u - p). \end{cases} \tag{16}$$

are analyzed in the $\tau = \varepsilon$ regime. For both equations, the $\tau = 0$ case is well-known as it recovers either the BBM equation for (15) or the Leray regularization [2] for (16). A combination of both can be put in connection with peridynamics [3].

Neither of these models are satisfying as they, despite some merits, fail at replicating desirable inviscid and viscous Burgers properties. Nevertheless, a modified version of (15), namely

$$\begin{cases} u_t + p_x = 0, \\ \qquad \tau p_t = \varepsilon^2 p_{xx} + \left(\frac{1}{2}u^2 - \varepsilon u_x - p\right), \end{cases} \tag{17}$$

which turns out to be equivalent to the viscoelastic Burgers model $\varepsilon u_{tt} - \varepsilon^2 u_{txx} + u_t + u u_x = \varepsilon u_{xx}$, has a desirable Kuznetsov-type approximation property, albeit with respect to the L^2 norm not the customary L^1.

Last but not least, we compare the results concerning (17) with the recently studied in [5] and [4]

$$\begin{cases} u_t + u u_x + \varepsilon^2 p_x = 0, \\ \qquad -\varepsilon^2 p_{xx} + p = \frac{1}{2}u_x^2. \end{cases} \tag{18}$$

[1] M. Colombo, G. Crippa, and L. V. Spinolo, *On the singular local limit for conservation laws with nonlocal fluxes*, Arch. Ration. Mech. Anal. **233** (2019), no. 3, 1131–1167.

[2] H. S. Bhat and R. C. Fetecau, *A Hamiltonian regularization of the Burgers equation*, J. Nonlinear Sci. **16** (2006), no. 6, 615–638.

[3] Q. Du, J. R. Kamm, R. B. Lehoucq, and M. L. Parks, *A new approach for a nonlocal, nonlinear conservation law*, SIAM J. Appl. Math. **72** (2012), no. 1, 464–487.

[4] Guelmame, B., and Houamed, H. (2024). Convergence rate for a regularized scalar conservation law. *Zeitschrift für angewandte Mathematik und Physik*, 75(3), 111.

[5] Guelmame, B., Junca, S., Clamond, D., and Pego, R. L. (2024). Global weak solutions of a Hamiltonian regularised Burgers equation. *Journal of Dynamics and Differential Equations*, 36(2), 1561–1589.

Kermack-McKendrick type models for epidemics with nonlocal aggregation terms

Ghaderi Z., Fatemeh

University of L'Aquila, Italy

Monday, May 25, 17:40-18:05, M17.81 – 8.019

We propose an approach to model spatial heterogeneity in SIR-type models for the spread of epidemics via *nonlocal aggregation terms*. More precisely, we first consider an SIR model with spatial movements driven by nonlocal aggregation terms, in which the inter-compartment and intra-compartment interaction terms are distinct, and modelled through smooth interaction kernels. For the Cauchy problem of said model we provide a full well-posedness theory on $\mathbb{R}^2$ for $L^1 \cap L^\infty \cap H^1$ initial conditions. The existence part is achieved by considering an approximated model with artificial linear diffusion, for which existence and uniqueness is proven via Duhamel's principle and Banach fixed point, and by providing suitable uniform estimates on the approximated solution in order to pass to the limit via classical compactness techniques. To prove uniqueness, we use classical L^2-stability which relies on the H^1-regularity of the solution. In the second part of the paper we provide a brief, general discussion on the steady states for these type of models, and display a specific example of non-trivial steady states for an SIS model with aggregations (driven by a single repulsive-attractive potential), the existence of which is determined by a threshold condition for a suitable "space-dependent" basic reproduction rate.

P12-2 Nonlocal Models

Singular limit for a class of nonlocal conservation laws via compensated compactness

De Nitti, Nicola; Coclite, Giuseppe Maria; Huang, Kuang

University of Pisa, Italy

Tuesday, May 26, 14:00-14:25, M17.02

We investigate macroscopic descriptions of traffic phenomena using fluid-dynamical models formulated as hyperbolic scalar conservation laws. In recent years, significant attention has been devoted to nonlocal variants of the classical Lighthill–Whitham–Richards (LWR) model, which account for drivers adjusting their velocity according to traffic conditions over a spatial range ahead. More precisely, we study the following class of nonlocal scalar conservation laws:

$$\partial_t u_\varepsilon + \partial_x\big(V(u_\varepsilon * \gamma_\varepsilon)\, u_\varepsilon\big) = 0,$$

where $u_\varepsilon : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ denotes the traffic density, $V : \mathbb{R} \to \mathbb{R}$ is a velocity function, $\varepsilon > 0$ is the nonlocal horizon parameter acting as a characteristic length scale, and $\gamma_\varepsilon(\cdot) := \varepsilon^{-1}\gamma(\cdot/\varepsilon)$ is a rescaled averaging kernel ($*$ denotes convolution with respect to the spatial variable). It is natural to assume that the velocity is non-increasing (i. e., higher traffic density leads to lower speed) and that the kernel is anisotropic, supported in $\mathbb{R}_-$, non-negative, and non-decreasing, reflecting that drivers adjust their speed according to an averaged downstream traffic density and assign greater weight to closer vehicles.

Our main result establishes that the averaged quantities $w_\varepsilon := u_\varepsilon * \gamma_\varepsilon$ converge strongly in $\mathrm{L}^1_{\mathrm{loc}}$ to the unique entropy solution of the local conservation law

$$\partial_t u + \partial_x(V(u)u) = 0,$$

as $\varepsilon \searrow 0$, i. e., as the kernel concentrates to a Dirac delta. The *nonlocal-to-local limit* plays a central role in establishing consistency between nonlocal and classical local traffic models.

Most previous results rely on total variation estimates, which limit their applicability to specific classes of kernels (namely, total variation may increase for anisotropic kernels that are not convex in $\mathbb{R}_-$). Our analysis circumvents any reliance on total variation bounds. Instead, we derive L^2-type estimates on the entropy production of w_ε and employ compensated compactness techniques. A key difficulty is the presence of a genuinely nonlocal dissipation mechanism.

Our convergence theorem applies to initial data in $\mathrm{L}^1 \cap \mathrm{L}^\infty$ and covers anisotropic kernels without convexity assumptions, thereby addressing a long-standing open problem. In particular, we treat both the piecewise constant kernel $\gamma := \mathbb{1}_{[-1,0]}$ coupled with the Greenshields affine velocity function and a class of strictly increasing kernels combined with non-increasing velocity functions.

Monotone-based numerical schemes for two-dimensional systems of nonlocal conservation laws

Beckers, Anika; Friedrich, Jan; Herty, Michael

RWTH Aachen, Germany

Tuesday, May 26, 14:25-14:50, M17.02

In this talk, we present numerical schemes for two-dimensional systems of conservation laws

$$\partial_t \rho^k + \mathrm{div}_{\mathbf{x}} \boldsymbol{f}^k \left(t, \mathbf{x}, \rho^k, \boldsymbol{\eta} * \boldsymbol{\rho}\right) = 0,$$

$k = 1, \ldots, K$, for the state variables $\boldsymbol{\rho} = \left(\rho^1, \ldots, \rho^K\right)$ depending on time $t \in \mathbb{R}^+$ and space $\mathbf{x} \in \mathbb{R}^2$. The flux is nonlocal in space, i.e. the solution evolves over time depending on the nonlocal terms $\boldsymbol{\eta} * \boldsymbol{\rho} \in \mathbb{R}^M$, which contain convolutions of the state variables with a mollifier. These couple the system weakly. The underlying flux function is rather general to guarantee that our results are applicable to many models, such as crowd movements, material flow on conveyor belts or cluster formation.

For the numerical approximation, we consider first-order schemes [1] that are based on adapting well-known monotone numerical flux functions after approximating the nonlocal terms. We state sufficient conditions to ensure the convergence of these monotone-based numerical schemes to the unique weak entropy solution. Moreover, we provide an error estimate that yields the convergence rate of $\mathcal{O}(\sqrt{\Delta t})$ for the numerical approximations of the solution. Our results include an existence and uniqueness proof of the nonlocal system, too.

Having established first-order accurate consistent numerical fluxes we use a CWENO reconstruction in space to derive higher-order numerical schemes. Applying a scaling limiter on the reconstruction ensures compliance with the maximum principle, including positivity preservation. However, the increased computational effort gives rise to challenges that must be addressed rigorously.

These theoretical findings are illustrated by numerical results. We will focus on a model for crowd movements, regarding the pedestrians as density. The interaction between pedestrians in the form of mutual repulsion is included in the model and causes the nonlocal dependence of the flux.

Acknowledgements. A. B. is funded by the German Research Foundation (DFG) through SPP 2410 'CoScaRa' under grant FR 4850/1-1 and partially supported by the DFG project 320021702/ GRK2326 'EDDy'.

[1] A. Beckers and J. Friedrich. Monotone-based Numerical Schemes for Two-Dimensional Systems of Nonlocal Conservation Laws, arXiv:2601.20494, 2026.

On the existence and uniqueness of nonlocal nonlinear conservation laws: a fixed point approach

Nick Zinat Matin, Hossein; Keimer, Alexander; Liverani, Lorenzo

Ecole Polytechnique, France

Tuesday, May 26, 14:50-15:15, M17.02

We investigate the well-posedness of scalar conservation laws whose flux depends both point-wise on the solution, and nonlocally, through integral averages. Our analysis is based on a fixed-point formulation, in which the nonlocal dependence is incorporated as a space–time dependent component of the flux, together with classical stability estimates for entropy solutions. This framework unifies and extends several models previously considered in the literature and applies, in particular, to conservation laws with memory effects arising from the nonlocal operator. We prove existence and uniqueness of weak entropy solutions. In addition, we present numerical simulations illustrating the effect of the interaction between local nonlinearities, nonlocal terms, and memory effects. Finally, we discuss the case in which the local dependence is linear, relating our results to the existing and essentially complete literature.

P13-1 Multiscale and Homogenization

Multiscale modelling of cancer invasion and metastasis: the effects of tumour heterogeneity

Katsaounis, Dimitrios; Sfakianakis, Nikolaos; Chaplain, Mark A.J.

RWTH University Aachen, Germany

Tuesday, May 26, 14:00-14:25, M17.17 – 1.017

Cancer invasion and metastasis are inherently multiscale processes driven by complex interactions between cancer cells and the tumour microenvironment [1]. A central mechanism driving cancer heterogeneity is the Epithelial-to-Mesenchymal Transition (EMT), a cellular programming process through which predominantly proliferative epithelial-like cancer cells (ECCs), forming the main bulk of solid tumours, progressively acquire migratory and invasive mesenchymal-like traits. Mesenchymal-like cancer cells (MCCs) are capable of actively invading surrounding tissue through the extracellular matrix and may disseminate to distant organs via the vasculature. Upon arrival at secondary sites, they can undergo the reverse Mesenchymal-to-Epithelial Transition (MET), enabling the initiation of metastatic growth. Importantly, EMT should be considered as a continuous process rather than a binary switch, giving rise to intermediate hybrid phenotypes, where ECCs progressively lose certain behavioural characteristics and adopt new capabilities that make them more aggressive and eventually becoming fully MCCs [2].

This invasion–metastasis cascade works across different scales, with large populations of ECCs (approx. 10^9 cells) coexisting with smaller populations of MCCs (approx. 10^3). As a consequence, classical modelling approaches that rely solely on either atomistic or macroscopic descriptions fail to adequately capture the full dynamics of tumour progression. Individual-based models are analytically intractable and computationally prohibitive at the scale of large tumour growth, while continuum macroscopic models often struggle to capture the dynamic behaviour of small populations of highly migratory cells [3].

In this talk, I will present a novel 3D hybrid multiscale mathematical model [4], that couples individual-based representations of migrating cancer cells with continuum descriptions of tumour growth, illustrating how EMT-driven phenotypic changes influence macroscopic invasion patterns within a multi-organ framework. Finally, I will introduce a phenotype-structured population model that incorporates continuous transitions along the epithelial–mesenchymal spectrum, providing a tractable framework for studying the emergence and maintenance of phenotypic heterogeneity in cancer.

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Multiscale hyperbolic-parabolic models for nonlinear reactive transport in heterogeneously fractured porous media

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University of Stuttgart, Germany

Tuesday, May 26, 14:25-14:50, M17.17 – 1.017

This talk addresses the asymptotic analysis of nonlinear reactive transport in a layered porous medium separated by a thin, geometrically heterogeneous fracture. The fracture is modeled as an ε-thin channel whose aperture and internal obstacle pattern vary periodically on the microscale. Transport in the bulk domains is governed by parabolic reaction–diffusion equations, while the fracture hosts a convection–diffusion–reaction process with nonlinear reactions on the walls and obstacle boundaries; notably, the transport inside the fracture is convection dominated with a Péclet number of order ε^{-1}.

Using multiscale asymptotics as $\varepsilon \to 0$, we derive a reduced homogenized model in which the thin fracture collapses to a flat interface. The homogenized limit consists of classical diffusion–reaction equations in the bulk domains coupled through homogenized nonlinear interface conditions and a first-order semilinear hyperbolic system posed on the flat interface. The latter encodes the effective fracture dynamics produced by the strong advection and the microscale geometry. The following aspects are addressed in this study: firstly, the well-posedness of the homogenized system is proven, including the derivation of regularity results; secondly, an explicit multiscale approximation incorporating boundary-layer correctors is constructed; and thirdly, quantitative error estimates in appropriate energy norms are established. These three aspects rigorously justify the reduced model and quantify the approximation accuracy with respect to the small parameter ε.

Numerical homogenization of multiscale hyperbolic conservation laws

Merlini, Giulia; Le Bris, Claude ; Legoll, Frédéric

CERMICS, ENPC and MATHERIALS, INRIA, France

Tuesday, May 26, 14:50-15:15, M17.17 – 1.017

This work deals with the prototypical multiscale hyperbolic conservation law

$$\partial_t u^\varepsilon(x,t) + \operatorname{div} F\left(\frac{x}{\varepsilon}, u^\varepsilon(x,t)\right) = 0 \tag{19}$$

where u^ε is a scalar-valued function and the flux F is a vector-valued function, highly oscillatory at the small scale ε. Accurate numerical approximation typically requires a spatial discretization finer than ε, leading to prohibitive computational costs. From the viewpoint of asymptotic analysis, some classes of equations, for example linear ergodic transport, admit a classical homogenization limit (i.e., the homogenized equation is again in the form of a conservation law), while in other cases such as *shear* transport, problems converge to a *delayed* equation [1,3]. In general, the limit obtained remains multiscale in nature. It is also well known that as soon as the original hyperbolic equation is slightly perturbed by a viscous (parabolic) term, then this vanishing viscosity term creates a classical homogenized limit, with effective parameters defined in terms of a corrector function, solution to an elliptic cell problem encoding the fine-scale structure of the flux [2].

Based on that observation, we have considered a strategy aiming at efficiently computing an approximation of the numerical solution given by a fine-scale finite volume scheme applied to

the original hyperbolic multiscale conservation law. The idea is to homogenize the numerical scheme discretizing the PDE, rather than the PDE itself, by exploiting the numerical viscosity naturally present in the numerical scheme. This strategy has been first put in practice for the classical Lax–Friedrichs scheme. We assume the flux $F(y, v)$ to be Y-periodic with respect to $y \in Y = [0, 1]^d$. We derive the equivalent equation associated to the scheme and, by means of a two-scale Ansatz, we obtain a well-posed cell problem in Y, that allows to compute the effective flux of the homogenized limit equation. The solution of the corrector problem depends on the fine discretizing parameters from the Lax-Friedrichs scheme.

The strategy has been implemented in a linear setting, with *well-prepared* initial conditions, following [2]. We will discuss numerical results showing that the resulting homogenized model provides an accurate and computationally efficient approximation of the reference solution in one and two space dimensions for a broad class of multiscale conservation laws.

Acknowledgements. The work is partially supported by the European Office of Aerospace Research & Development, under Grant FA8655-24-1-7057.

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Modeling edema dynamics in soft tissue using mixture theory

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Tuesday, May 26, 15:15-15:40, M17.17 – 1.017

Tissues are formed by a mixture of cells, liquid in which chemical substances are dissolved, and an extracellular matrix made of polymer threads that keep the cells bound together. When tissues are damaged due to mechanical or chemical causes, they react by sending a fluid flow to the part involved. The uncontrolled buildup of fluid in the involved tissue leads to edema, which, if it is not drained, affects tissue function and consequently human health. Tissues can be represented as a mixture [3, 4] with three fluid components [2] consisting of a biomass phase (cells and polymeric matrix), which is split into two components - alive and dead cells, with volumetric fractions $\phi_{\mathrm{a}}(\mathbf{x}, t)$ and $\phi_{\mathrm{d}}(\mathbf{x}, t)$ respectively, and a fluid component, $\phi_{\mathrm{f}}(\mathbf{x}, t)$, that carries nutrients, waste, and toxicants.

The mixture [1] is governed by the mass and linear momentum balance laws coupled with advection-reaction-diffusion equations for the concentrations of nutrients $c_{\mathrm{n}}(\mathbf{x}, t)$ and toxicants $c_{\mathrm{p}}(\mathbf{x}, t)$. In our paper we develop and simulate a model, by using a the open acces freeFEM platform [5] that solves systems of partial differential equations through finite element methods. In particular, the effects of toxicants on tissue, the resulting fluid infiltration, and its drainage by medicine treatment are investigated.

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P14-1 Riemann Problems and Wave Patterns

A convergent front tracking scheme with an application to gas dynamics

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Monday, May 25, 14:00-14:25, M17.81 – 8.019

We present a modified Front Tracking (mFT) scheme for hyperbolic systems of conservation laws in one space dimension, in which we allow arbitrarily large nonlinear waves. We build the scheme by introducing and solving a "generalized Riemann Problem", which yields exact solutions for finite times. After construction of the scheme, under reasonable assumptions, we show that the mFT approximations converge to a weak* solution of the system. This essentially reduces existence of solutions with large amplitude data to obtaining uniform bounds on the total variation of the approximations.

We apply the modified Front Tracking (mFT) scheme to the equations of gas dynamics. We solve the generalized Riemann Problem and define the scheme for both full and isentropic systems, and show the equivalence of the Eulerian and Lagrangian frames. For the p-system (isentropic gas dynamics in a Lagrangian frame), we show that there is no finite accumulation of interaction times, if there is a nonincreasing potential on the approximate solutions constructed through our scheme.

This is joint work with Robin Young (University of Massachusetts Amherst).

An exact Riemann solver for two layer two-phase flow model with Chaplygin pressure law

Thawait, Sheetal Prasad

Indian Institute of Technology Kharagpur, Kharagpur, India

Monday, May 25, 14:25-14:50, M17.81 – 8.019

In this paper we investigate the Riemann problem associated with hyperbolic system of partial differential equations arising in the modeling of laminar stratified two-phase flow featuring distinct phase velocities where the pressure is governed by the Chaplygin pressure law. We derive rarefaction wave curves by using associated Riemann invariants and shock waves using the Rankine-Hugoniot condition. By enforcing the constancy of averaged velocity and total density across contact discontinuity, we convert the governing system of partial differential equations into a system of nonlinear algebraic equations, which we solve precisely. Further, we determine the complete wave structure of the two phase flow model accurately. To evaluate the robustness of solver, we investigate a variety of potential test cases depending on the wave configuration. To confirm our analytical solution, we also apply three Godunov-type finite volume schemes, whose solutions is fully numerical, not dependent on the suggested Riemann solver.

Exact Riemann solver for a laminar two-phase flow model with two velocities

Raja Sekhar, T

Indian Institute of Technology, Kharagpur, India

Monday, May 25, 14:50-15:15, M17.81 – 8.019

In this work, we investigate the Riemann problem for a quasilinear hyperbolic system of partial differential equations arising in a laminar two-phase flow model with two velocities. Rarefaction and shock waves are constructed explicitly using Riemann invariants and the Rankine-Hugoniot conditions, respectively. By exploiting the continuity of the averaged velocity and the total density, the governing system is reduced to a system of nonlinear algebraic equations, which is solved exactly. The complete wave structure of the two-layer two-phase flow is then fully characterized. Several test cases, corresponding to different wave configurations, are examined to demonstrate the robustness of the proposed Riemann solver. Furthermore, various Godunov-type finite volume schemes whose solutions are independent of the proposed solver are implemented to validate the analytical results.

A Riemann problem in two-phase real fluid mixture

Kumar, Pardeep; Rosen, Patricio

Centrum Wiskunde & Informatica, Amsterdam, The Netherlands

Monday, May 25, 15:15-15:40, M17.81 – 8.019

Pipeline transport of dense-phase CO_2-rich mixtures is a crucial component in carbon capture and storage (CCS). Accurate modeling requires coupling of fluid dynamics and thermodynamics, especially during transient events such as depressurization. In this work, we present a unified framework for two-phase multicomponent transport in pipelines that integrates both aspects. Specifically, we employ the homogeneous equilibrium model (HEM) for modeling the transport of two-phase CO_2-rich mixture, with thermodynamic closure provided by a Helmholtz energy-based equation of state. The depressurization process is formulated as a Riemann problem in a two-phase real-fluid mixture. Phase equilibrium calculations are performed using UVN-flash, supplemented with a stability analysis procedure to detect phase separation and generate initial guesses for the phase-equilibrium calculations. Specifically, we introduce a novel tailored UVN-flash routine that aligns with the fluid dynamics formulation. This is achieved by introducing an alternative and better-scaled set of variables for the phase-equilibrium calculations. The proposed framework is applied to the depressurization of pipelines containing CO_2-rich mixtures, demonstrating its effectiveness for CCS–relevant applications.

P14-2 Riemann Problems and Wave Patterns

Do Riemann solutions consist of rarefactions, shocks, and constants?

Plohr, Bradley J.; Schecter, Stephen; Marchesin, Dan

Los Alamos, United States of America

Tuesday, May 26, 14:00-14:25, M17.81 – 8.019

A Riemann solution for a strictly hyperbolic system of conservation laws is traditionally constructed from rarefaction and shock waves separated by constant states. Does a Riemann solution have this structure when it is only assumed to be L^∞?

Rarefaction waves are continuous portions in a solution whereas shock waves are jump discontinuities. Identifying these features in an L^∞ solution requires a notion of continuity that respects equality almost everywhere. Using the concept of essential image, we ascribe pointwise accumulation sets to members of L^∞. We find that the system of conservation laws imposes strong regularity on the essential-image accumulation sets of a Riemann solution.

We prove that if a solution is continuous in the essential-image sense and satisfies the standard resonance condition in an open interval, then it is a bounded variation rarefaction wave, being a re-parametrization of a rarefaction integral curve by speed, although it might fail to be genuinely nonlinear. We also characterize an essential-image discontinuity: it can have more than the usual pair of accumulation values, but the Rankine–Hugoniot condition limits their number. In general, the wave structure in a Riemann solution can be anomalous, but we prove that if there are finitely many discontinuities, a Riemann solution is composed of constant states and mild generalizations of rarefaction and shock waves.

Viscosity-dependent wave patterns in the Riemann problem for nonstrictly hyperbolic conservation laws

Pellhammer, Valentin

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Tuesday, May 26, 14:25-14:50, M17.81 – 8.019

For nonstrictly hyperbolic systems of conservation laws, the admissibility of shock waves may depend on the assumed background viscosity through the viscous-profile criterion. As a consequence, the Riemann solver itself can depend on the viscosity in the underlying regularization. In this talk, I discuss this phenomenon for a class of quadratic conservation laws and show that varying the background viscosity may lead to different admissible wave patterns for the same Riemann data. This viscosity dependence affects the structure of the wave curves and the resulting self-similar solutions of the Riemann problem. A key role is played by heteroclinic bifurcations in the associated profile equations, which mark transitions between different families of admissible shocks. I will then relate these results to magnetohydrodynamics, where viscosity-dependent shock admissibility is well known and where the quadratic models considered here arise as local approximations.

Maximal energy dissipation principle and the Riemann problem for shallow water flow

Ružičić, Sanja; Daw, Dalal; Nedeljkov, Marko

University of Novi Sad, Serbia

Tuesday, May 26, 14:50-15:15, M17.81 – 8.019

The shallow water system with a single discontinuity of the bottom topography is a simple model of a fluid flow with a sudden change in bottom elevation. The discontinuity is modeled by an increasing step function. Using a piecewise continuous approximation with the parameter $\varepsilon \ll 1$ of the step function at $x = 0$, the Riemann problem for the system is transferred into the minimization of energy problem with the constraints: The approximated solution has to satisfy the Dafermos's maximal dissipation condition around the discontinuity, while shock and rarefaction waves must have non-positive velocity for $x < 0$ and non-negative one for $x > 0$. The application of the procedure is made for the dam-break problem with the additional assumptions that fluid is moving from left to right for $x < 0$ and the bed is dry for $x > 0$. The existence of an admissible solution corresponds to the dam failure. A shadow wave is a perturbation that consists of two shock waves with an infinitesimally small area between. It is used to model the fluid behavior near the discontinuity. The procedure given here is also applicable in the general case when bed is not dry for $x > 0$.

P14-3 Riemann Problems and Wave Patterns

Formation of vacuum state and delta shock in the solution of two-dimensional Riemann problem for zero pressure gas dynamics

Pandey, Anamika

Indian Institute of Technology, Kharagpur, India

Tuesday, May 26, 16:00-16:25, M17.81 – 8.019

The mathematical modeling of various natural phenomena often relies on systems of nonlinear hyperbolic conservation laws. Among these, the Euler equations play a central role in describing the fundamental conservation laws of mass, momentum, and energy. However, due to the inherent complexity of the full Euler system, researchers frequently turn to simplified models that preserve essential dynamics while offering greater tractability. One such model is the pressureless Euler system, which effectively captures the behaviour of freely moving particles that coalesce upon collision. This system has found widespread applications, ranging from modeling large-scale structure formation in cosmology to simulating dusty or particulate media where pressure effects are negligible.

In this work, we study the two-dimensional pressureless Euler system with conservation of total energy

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0,\\
&\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho u v)}{\partial y} = 0,\\
&\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho u v)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} = 0,\\
&\frac{\partial}{\partial t}\left(\rho e + \frac{\rho}{2}(u^2+v^2)\right) + \frac{\partial}{\partial x}\left(\left(\rho e + \frac{\rho}{2}(u^2+v^2)\right)u\right) + \frac{\partial}{\partial y}\left(\left(\rho e + \frac{\rho}{2}(u^2+v^2)\right)v\right) = 0,
\end{aligned} \tag{20}$$

where the variable $\rho \geq 0$ denotes the density, u, v represent x- and y-velocity components and $e \geq 0$ denotes specific energy, respectively. System (20) arises by setting the pressure $p = 0$ in the 2-D compressible Euler system. This formulation extends the classical pressureless gas dynamics model by incorporating conservation of total energy. Although the pressure term vanishes, the energy equation describes the transport and possible concentration of energy along singular structures such as delta shocks. Such a formulation is relevant in applications including astrophysical structure formation and the dynamics of granular or particulate flows, where aggregation processes occur while pressure effects remain negligible.

We investigate the associated two-dimensional Riemann problem with three-piece constant initial data. The analysis focuses on nonlinear wave interactions with three-piece constant Riemann initial data, particularly those involving contact discontinuities and delta shocks. A key feature of the solution is the emergence of delta shock waves, characterized by Dirac delta distributions in both the density and internal energy variables. By employing generalized characteristic analysis, we identify nine topologically distinct solution configurations. Some of these exhibit intriguing phenomena analogous to Mach reflection-type patterns, while others lead to the development of vacuum regions, illustrating the rich multidimensional structure of solutions. To validate our analytical findings, we conduct high-resolution numerical simulations using a semidiscrete central upwind scheme. The numerical results show excellent agreement with the theoretical predictions, confirming the accuracy of our analysis.

Generalised Riemann problem and wave interactions in a macroscopic production model

Sahu, Lilu

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Tuesday, May 26, 16:25-16:50, M17.81 – 8.019

In this work, we consider a hyperbolic system of balance laws which represents the macroscopic production model that describes high-volume production flows:

$$\begin{cases} \rho_t + (\rho v)_x = 0, \\ (\rho v(1+k\rho))_t + (\rho v^2(1+k\rho))_x = \alpha\, \rho v(1+k\rho), \qquad x \in \mathbb{R},\ t > 0 \end{cases}$$

where the dependent variables v and ρ are the velocity and product density, respectively, which describe the progress of work at the production stage x and the time t. First, we demonstrate that the governing system is compatible with a set of differential constraints, which enables us to handle the non-homogeneous term and makes the system diagonizable. Further we obtain the solution, which consists of shocks and rarefaction waves, of the Riemann problem as well as the generalised Riemann problem analytically. Then we analyse all possible interactions of elementary waves of the Riemann problem. Here, the main difficulty is to identify the solution of the new Riemann problem which is formed at the interaction of shock and rarefaction due to the presence of non-homogeneous term. However, using the idea of the generalised Riemann problem, we are able to obtain solutions up to a certain time t.

Dynamics of delta shock waves originating from a three pieces Riemann problem for traffic flow model

Priyanka; M. Zafar

Department of Mathematics and Computing, Dr. B R Ambedkar NIT Jalandhar, India

Tuesday, May 26, 16:50-17:15, M17.81 – 8.019

This article addresses the interactions of delta shock waves with elementary waves in a strictly hyperbolic system of conservation laws derived from a traffic flow model. On utilizing the technique of splitting delta function, the global structures of solutions are constructed and analyzed one by one. Moreover, we prove that the solutions of the Riemann problems are stable under the small perturbations of the Riemann data.

Note: This paper has also been published in "Mathematical Methods in the Applied Sciences".

Publication Detail: Priyanka and M. Zafar, *Dynamics of Delta Shock Waves Originating From a Three-Piece Riemann Problem for Traffic Flow Model,* **Mathematical Methods in the Applied Sciences (Wiley)** (2025), DOI: https://doi.org/10.1002/mma.70050

P14-4 Riemann Problems and Wave Patterns

On the characteristic fields of averaged models of two phase flows

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Thursday, May 28, 14:00-14:25, M17.81 – 8.019

We investigate some original mathematical properties shared by the two-phase flow models [1, 2] obtained from averaging the Euler equations of fluid dynamics [3]. The focus will be on three one-pressure, two-velocity models of increasing size: the 3-equation drift model, the 4-equation drift model, and the 6-equation two-fluid model. One original property of these models is the non-trivial characterization of the hyperbolicity domain, which may be neither open nor simply connected. A second original property is the non-trivial structure of the characteristic fields, which may be neither ordered, Genuinely Nonlinear (GNL), Linearly Degenerate (LD). A third shared property is that the mixture sound speed can be significantly lower than the phase sound speeds. A final example of an original property with significant impact on the design of numerical methods is the presence of sonic points in low-Mach-number regimes.

Unlike the Euler equations, the characteristic fields of these models display a non-trivial structure when the two velocities are distinct ($\vec{u}_1 \neq \vec{u}_2$). This originates from the nonlinearity arising from the coupling between the two velocities, which corresponds to original physical configurations that challenge numerical methods based on a simple linearization of the system—as is typically done in single-phase flows (e.g., upwind schemes [4]).

In the case of mechanical equilibrium ($\vec{u}_1 = \vec{u}_2$), all three models are hyperbolic with characteristic fields that are either Genuinely Nonlinear (GNL) or Linearly Degenerate (LD), as in the Euler equations for perfect gas dynamics. For the 3-equation two-phase model, however, establishing this hyperbolicity result is particularly challenging because, unlike the Euler equations, it is not possible to decouple the entropy wave from the other waves.

In the case of mechanical disequilibrium ($\vec{u}_1 \neq \vec{u}_2$), we assume both phases to be incompressible—a first-order approximation justified in low Mach flows regimes. The incompressible versions of the 4- and 6-equation models facilitate a better understanding of the nonlinearities associated with mixture composition waves, which are specific to two-phase flows. As expected from the theory [5], solving the Riemann problem then reveals composite waves and there can be more waves than equations. We will explain why these non-classical composite waves are essential for correctly describing the flow, especially in separation configurations where the two phases move in opposite directions ($\vec{u}_1 \cdot \vec{u}_2 < 0$).

This analysis of the models has a significant impact on the design of numerical methods used for their simulation. We illustrate the pitfalls associated with a direct transposition of the numerical methods originally developed for single-phase flows and give some guidelines to improve existing softwares.

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Convergence to superposition of boundary layer, rarefaction and shock for the inflow problem of the 1D Navier–Stokes equations

Oh, HyeonSeop; Han, Sungho; Kang, Moon-Jin; Kim, Jeongho; Kim, Nayeon

Korea Advanced Institute of Science and Technology (KAIST), South Korea

Thursday, May 28, 14:25-14:50, M17.81 – 8.019

In this talk, we study the asymptotic stability of solutions to the inflow problem for the one-dimensional isentropic Navier–Stokes system in the half space. When the boundary value lies in the subsonic region, all possible asymptotic wave patterns were classified in [1]. We consider the most complicated case, the superposition of a boundary layer, a rarefaction wave, and a viscous shock wave. In this superposition, the boundary layer is degenerate and large. We prove that if the initial perturbation and the strengths of the rarefaction wave and the shock wave are small, then the solution converges asymptotically to the superposition up to a dynamical shift of the shock. This talk is based on a joint work with Sungho Han, Moon-Jin Kang, Jeongho Kim, and Nayeon Kim [2].

[1] A. Matumura. Inflow and outflow problems in the half space for a one-dimensional isentropic model system of compressible viscous gas. *Methods and Applications of Analysis*, 8(4):645–666, 2001.

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A local-in-time quadratic functional for scalar conservation laws

Baitenov, Egor; Modena, Stefano

Gran Sasso Science Institute, Italy

Thursday, May 28, 14:50-15:15, M17.81 – 8.019

We construct a new Lyapunov functional for scalar conservation laws, which is quadratic (w.r.t. the total variation of the initial datum), it controls the change in speed of waves in wavefront tracking approximate solutions, and, differently than functionals with similar properties already present in the literature, it is purely local in time, meaning that it depends only on the present profile of the (approximate) solution (and not on its past or future evolution).

Effects of quartic nonlinearity on shock wave propagation in non-ideal media

Neelam

National Institute of Technology Warangal, India

CANCELLED

This study examines shock wave propagation in a medium exhibiting mixed-type weak nonlinearity. The flow is governed by one-dimensional Euler equations supplemented with the van der Waals equation of state. During the flow, a change in the sign of the fundamental derivative causes the appearance of quartic nonlinearity that leads to the coexistence of compression and expansion waves within a single pulse. An analytical investigation of wave interactions reveals unconventional phenomena, including precursor wave formation and shock splitting. The influence of non-ideal fluids on wave evolution and interaction is analyzed using van der Waals parameters.

P14-5 Riemann Problems and Wave Patterns

Self-similar analysis of strong shock waves in a magnetized, non-ideal, self-gravitating gas with finite electrical conductivity

Yadav, Sanjay; Yadav, Priyanshu

Central University of Rajasthan, India Alliance University, Bangalore, India

Thursday, May 28, 16:00-16:25, M17.81 – 8.019

In this paper, we investigate the propagation of strong shock waves in a magnetized, non-ideal, self-gravitating gaseous medium by incorporating the effects of finite electrical conductivity. Unlike classical ideal magnetohydrodynamic models, the present formulation accounts for magnetic diffusion arising from resistive effects, which are particularly relevant in dense astrophysical plasmas and partially ionized environments. The governing equations of motion are formulated for one-dimensional unsteady flow under cylindrical and spherical symmetries, including non-ideal gas behavior, self-gravity, and an azimuthal magnetic field with finite magnetic diffusivity. The inclusion of resistive effects provides a more realistic description of shock propagation in astrophysical and laboratory plasmas, such as supernova remnants, protostellar environments, and exploding wire experiments. The present study extends classical self-similar shock models and offers new insights into the coupled roles of resistivity, non-ideal thermodynamics, and gravity in magnetogasdynamic shock phenomena.

Topological structures in systems of conservation laws

Lambert, W.J.; Eschenazi, C.S.; López-Bambarén, E. P; López-Flores, M.M.;

Marchesin, D.; Palmeira, C.F.B.

Fed. Univ. of Alfenas

Thursday, May 28, 16:25-16:50, M17.81 – 8.019

We present a topological framework for analyzing systems of conservation laws, focusing on a system of two equations with quadratic flux functions that possesses an elliptic region in state space. Central to our approach is the *wave manifold* $\mathcal{W}$, a smooth three-dimensional manifold in which each point represents a shock wave satisfying the Rankine–Hugoniot conditions.

To facilitate explicit computation and visualization, we introduce adapted coordinates (z, τ, Y) on $\mathcal{W}$. In these coordinates, we derive closed-form expressions for the fundamental geometric structures: • the *characteristic surface* C, which is the natural domain for rarefaction curves and is divided into a slow part C_s and a fast part C_f; • the *sonic surfaces* Son and Son$'$, where the shock speed attains an extremum along Hugoniot or Hugoniot$'$ curves; • the *coincidence curve* $\mathcal{E}$, which lies at the intersection of C with the projection of the elliptic region and along which Hugoniot curves are tangent to C; • the *sonic fold*, the locus where Hugoniot curves are tangent to the sonic surface, which describes the saturation of hysteresis; • the *secondary bifurcation lines* $\mathcal{B}$ and $\mathcal{B}'$, where the Hugoniot and Hugoniot$'$ foliations become singular; • the *double-sonic locus* $\mathcal{D}$, where the surfaces Son and Son$'$ intersect transversely; and • various extensions of the elliptic region within $\mathcal{W}$, such as the saturation surfaces SCC and SCC'.

Within this geometric setting we construct *wave curves*. Each wave curve is built by concatenating arcs of: rarefaction curves (integral curves of a line field defined on C), shock curves (segments of Hugoniot or Hugoniot$'$ curves satisfying appropriate admissibility conditions), and composite curves (obtained by lifting rarefaction curves from C to Son$'$). Projecting these structures onto the characteristic manifold yields a natural subdivision of C_s (and analogously C_f) that organizes the Riemann data according to the qualitative structure of the associated wave

curve. We also study *non-local shocks* and identify wave curves that exhibit gaps—i.e., intervals where the shock curve is not continuous.

To solve a Riemann problem with left state W_L and right state W_R, we first lift these states to points $\mathcal{U}_L \in C_s$ and $\mathcal{U}_R \in C_f$ in the wave manifold. From $\mathcal{U}_L$ we construct the slow wave curve and generate its associated *intermediate surface*—the union of all Hugoniot′ curves emanating from points along this slow wave curve. Simultaneously, we construct the fast wave curve starting from $\mathcal{U}_R$ and apply the reflection operator (which exchanges left and right states). The intersection of the reflected fast wave curve with the intermediate surface determines a point whose corresponding right state serves as the intermediate constant state in the Riemann solution. The full solution is then assembled as a slow-wave group (connecting W_L to the intermediate state) followed by a fast-wave group (connecting the intermediate state to W_R).

We demonstrate this topological solution procedure through representative examples, providing graphical representations of the wave curves, intermediate surfaces, and their intersections within the three-dimensional wave manifold. These visualizations clarify how the characteristic, sonic, and coincidence surfaces organize the admissible wave patterns and enable a systematic construction of Riemann solutions. Our approach offers a transparent geometric interpretation of the classical wave-curve intersection method.

This work was supported by FAPEMIG under grant APQ-02561-25 and CNPq under grant 409718/2024-6.

Shock wave evolution and conservation laws in compressible flow via symmetry based analysis

Sharma, Prince; Arora, Rajan

Indian Institute of Technology Roorkee, India

Thursday, May 28, 16:50-17:15, M17.81 – 8.019

This work presents a Lie symmetry analysis of a one-dimensional system of hyperbolic balance laws governing the evolution of mass density, velocity, and pressure in a compressible continuous medium. The governing equations form a nonlinear system of partial differential equations representing the fundamental equations of motion. Using the classical Lie group method, the complete set of infinitesimal symmetry generators is determined and the corresponding Lie algebra structure is established. An optimal system of one-dimensional subalgebras is constructed through adjoint transformations, enabling systematic symmetry reductions of the original system. These reductions yield several classes of invariant solutions, providing analytical insight into nonlinear wave propagation and the underlying dynamical structure. Furthermore, the property of nonlinear self-adjointness is employed to derive conservation laws associated with physically significant quantities, including momentum and energy transport. The results contribute to the analytical characterization of nonlinear hyperbolic systems and demonstrate the effectiveness of symmetry-based methods for investigating wave dynamics and structural properties of compressible continuum models.

Riemann problem for a pressureless gas dynamics system with time-gradually-degenerate linear damping

De la cruz, Richard; Neves, Wladimir

Universidad Pedagógica y Tecnológica de Colombia, Colombia

Thursday, May 28, 17:15-17:40, M17.81 – 8.019

Balance laws with time-dependent coefficients arise naturally in several areas of continuum physics, including adhesion models and pressureless gas dynamics with damping effects. When the damping term gradually degenerates in time, the classical Riemann theory exhibits new qualitative phenomena that do not appear in autonomous or uniformly damped systems. This talk addresses the Riemann problem for a nonautonomous pressureless gas dynamics model subject to time-gradually-degenerate damping. It is shown how the degeneration of the damping term significantly affects wave interactions and leads to the emergence of delta shock waves [1]. The analysis is based on time-dependent similarity variables and nonlinear viscous approximations, which allow the explicit construction of Riemann solutions. An over-compressive entropy condition is identified, ensuring the uniqueness of delta shock solutions in this nonautonomous framework. These results highlight the delicate role of time-dependent damping in balance laws and provide insight into the mechanisms driving concentration phenomena beyond the classical autonomous setting.

[1] R. De la cruz and W. Neves, On a class of nonautonomous quasilinear systems with general time-gradually-degenerate damping, *Journal of Differential Equations* 416:52–81, 2025.

Interactions of delta shock waves in generalized Chaplygin gas with source term

Minhajul; Rakib, Mondal

Depaerment of Mathematics, BITS Pilani K K Birla Goa Campus, India

Thursday, May 28, 17:40-18:05, M17.81 – 8.019

In this talk, we consider the wave interactions for the one-dimensional generalized Chaplygin gas equations with a nonlinear time-dependent source term. We discuss the construction of the Riemann solution for this non homogeneous hyperbolic system and show that the solution contains a singularity under certain conditions in the initial data; that is, a δ-shock wave arises in the solution. Finally, we discuss the wave interactions involving the δ-shock wave to establish the global solution by taking three piece-wise constant perturbed initial data.

P15-1 Numerical Analysis

A posteriori error estimates for high order approximations of hyperbolic balance laws

Sikstel, Aleksey; Giesselmann, Jan; Guerra, Graziano

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Tuesday, May 26, 14:00-14:25, 17.92 – 9.019

We prove rigorous a posteriori error estimates for high order Runge-Kutta discontinuous Galerkin (RKDG) schemes approximating nonlinear systems of hyperbolic balance laws (i.e. conservation laws with source terms) in one spatial dimension. The estimates are based on [1] where stability results for ε-approximate solutions to systems of balance laws will be obtained, similar to the seminal work by Bressan, Chiri and Shen [2] on conservation laws.

Previous results for first-order finite volume schemes [3] for conservation laws are extended to high-order numerical methods. Our estimators rely on a new way to localize residuals and a novel method to compute dual norms of these local residuals. Computing dual norms becomes possible by a reconstruction of the RKDG solution continuously within space-time cells and a direct approximation of the localized residual. We show novel estimates for approximations via tensor-product polynomials and, in addition, consider computable bounds for contributions due to the source terms. This approach yields up-to-a-constant computable a posteriori upper bounds on the numerical error.

[1] G. Guerra and A. Sikstel. A posteriori estimates for systems of one-dimensional hyperbolic balance laws. in prepration, 03 2026.

[2] A. Bressan, M. T. Chiri, and W. Shen. A posteriori error estimates for numerical solutions to hyperbolic conservation laws. *Archive for Rational Mechanics and Analysis*, 241(1):357–402, July 2021.

[3] J. Giesselmann and A. Sikstel. A-posteriori error estimates for systems of hyperbolic conservation laws via computing negative norms of local residuals. *IMA Journal of Numerical Analysis*, page drae111, 03 2025.

A posteriori error analysis of a Runge-Kutta discontinuous Galerkin scheme for nonlinear convection-diffusion systems in convection-dominated flows

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Technical University of Darmstadt, Germany

Tuesday, May 26, 14:25-14:50, 17.92 – 9.019

We derive computable and reliable a posteriori upper bounds for fully discrete Runge–Kutta discontinuous Galerkin approximations of the one-dimensional nonlinear convection–diffusion system $\partial_t u + \partial_x f(u) = \varepsilon\, \partial_x(A(u)\partial_x u)$ ($\varepsilon \geq 0$), under a strictly convex entropy/entropy-flux structure. We treat a broad class of one-dimensional systems, with particular emphasis on convection-dominated ($\varepsilon \ll 1$) and degenerate parabolic problems where diffusion may act only on selected components. Our a posteriori error bounds are valid for a family of discontinuous Galerkin spatial discretizations and various temporal discretizations that include explicit and implicit–explicit time-stepping schemes, popular tools for practical simulations of this class of

problem. We prove that our estimators provide reliable upper bounds for the error of the numerical method and present numerical evidence showing that they achieve the same order of convergence as the error. With emphasis on the convection-dominated regime, we explicitly track the estimator dependence on viscosity, including the limit $\varepsilon \to 0$.

Error estimates of an asymptotic-preserving discontinuous Galerkin method for the continuous data assimilation algorithm of Navier-Stokes system

Anandan, Megala ; Lukáčová-Medviďová, Mária; Zhang, Jiahui

Johannes Gutenberg Universität Mainz, Germany

Tuesday, May 26, 14:50-15:15, 17.92 – 9.019

Continuous data assimilation (CDA) algorithms incorporate observational measurement data, predicted continuously in time from physical systems, as a feedback (or nudging) mechanism to accurately reconstruct the current state and predict the future evolution of the system. In this talk, we discuss an asymptotic-preserving discontinuous Galerkin (DG) method for the synchronised weakly-compressibe Navier-Stokes system arising in the continuous data assimilation. We establish energy stability and derive a priori estimates for the semi-discrete DG scheme, prove its consistency, and obtain error estimates between the DG numerical solution and the (local-in-time unique strong) solution of the synchronised weakly-compressible Navier-Stokes system. Combining these results with the analytical results, we conclude that the numerical solutions obtained by a DG method for the synchronised weakly-compressible Navier-Stokes system approximate a weak solution of the observed weakly-compressible Navier-Stokes system. Consequently, we can predict the values of the observed solution by taking into account its proximity to the synchronised system.

Acknowledgements. Theoretical aspects of data assimilation are studied in cooperation with E. Feireisl (Prague).
This research was funded by the Gutenberg Research Fellowship, TRR146 Multiscale simulation methods for soft matter systems, and by DAAD-DST personell exchange programme.

Numerical Methods for the Camassa-Holm equation in the zero-filter limit

Geladaris, Dimitrios; Rohde, Christian

Germany

Tuesday, May 26, 15:15-15:40, 17.92 – 9.019

Over the past decades, the Navier–Stokes–α (NS–α) model has attracted attention as a regularized model for incompressible turbulence, where an energy penalty suppresses the formation of structures below a characteristic length scale α (see [1]). The Camassa–Holm equation can be viewed as a one-dimensional model problem capturing essential features of the NS–α dynamics and provides a useful setting for the development and analysis of numerical methods (see [2]).

In our work, we study finite difference schemes for the Camassa–Holm equation with general filtering parameter $\alpha > 0$. First, building on earlier energy-stable finite-difference schemes developed for the case of $\alpha = 1$ (see [3]), we extend the non-conservative approach to govern filtering parameters α in the range $(0, 1]$. At the expense of a stricter time-step condition we can achieve similarly as in [3] a discrete total energy inequality.
Second, we introduce a new conservative finite difference scheme and show that it is asymptotic preserving in the zero-filter limit $\alpha \to 0$ using the time-step restriction of the limit model.

We conclude with an extensive comparison of the numerical methods and a discussion on the generalization of the approaches to the NS–α model.

[1] C. Foias, D. Holm, and E. Titi. The Navier–Stokes-α model of fluid turbulence. *Physica D*, 152–153:505–519, 2001.

[2] R. Camassa and D. Holm. An integrable shallow water equation with peaked solitons. *Physical Review Letters*, 71:1661–1664, 1993.

[3] G. M. Coclite, K. H. Karlsen, and N. H. Risebro. An explicit finite difference scheme for the Camassa–Holm equation. *Advances in Differential Equations*, 13(7–8):681–732, 2008.

P15-2 Numerical Analysis

Convergence of a finite volume scheme for the Navier-Stokes-Korteweg equations to dissipative solutions

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Tuesday, May 26, 16:00-16:25, 17.92 – 9.019

The Navier-Stokes-Korteweg (NSK) system of equations extends the Navier-Stokes system to model capillarity effects in fluids. We propose a simple semi-discrete finite volume scheme to numerically solve the NSK system on a cartesian grid with periodic boundary conditions which has a novel combination of properties: It conserves mass and momentum, is energy dissipative, and operates directly on the NSK system without employing hyperbolic approximation or auxiliary variables. We present a proof of convergence of this scheme to dissipative solutions in the spirit of [1], as well as a weak-strong uniqueness result using relative energy methods.

Acknowledgements. All authors were supported by the DFG within the priority research program SPP 2410, project 525866748.

[1] E. Feireisl, M. Lukáčová-Medviďová, H. Mizerová, and B. She, *Numerical analysis of compressible fluid flows*, vol. 20 of MS&A, Model. Simul. Appl., Cham: Springer, 2021, https://doi.org/10.1007/978-3-030-73788-7.

[2] J. Giesselmann, P. Öffner, and R. Sauerborn. A structure preserving finite volume scheme for the Navier-Stokes-Korteweg equations, 2026. arXiv:2601.08498 [math.NA] https://doi.org/10.48550/arXiv.2601.08498

Convergence of a discretized observer for the Euler equations

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Technical University of Darmstadt, Germany

Tuesday, May 26, 16:25-16:50, 17.92 – 9.019

The aim of data assimilation is to incorporate available measurement data into an underlying physical model of some system. Often, only part of a system's state is measured. For example, for the Euler equations, flow velocities can be measured relatively easily using particle image velocimetry, while densities are challenging to measure directly. The main question is then, if it is possible to reconstruct the full state from the measured data.

In [1], a distributed Luenberger-type observer is presented for the barotropic Euler system on the continuous level. This approach has a *nudging* parameter, that controls how strongly the velocity of the observer system is forced towards the measurements. Exponential *synchronization*, that is, convergence of solutions of the observer system towards the original system state, is shown.

To use the observer, one must solve it numerically. It is then required to understand the convergence of the discretized observer and the influence of discretization errors, particularly for long times. We prove an error estimate that compares the original system state with the numerical solution of the observer system. The error bound consists of two parts: the first part decays exponentially with time and depends on the difference in the initial state of the observer

and the original system. The second part is proportional to the discretization parameters, with a constant independent of time, depending only on the original system state.

In this contribution, we use the structure-preserving discretization from [2]. For the nudging term, we use the linear interpolant of the measured data. This treatment is similar to the ideas in [3], where discrete measured data is interpolated to nudge the system on the continuous level.

We present a convergence analysis for the fully discrete observer using relative entropy techniques. Using the stabilizing effect of the nudging term, we prove the long-time stability of the discrete observer for sufficiently smooth solutions.

If time permits, we comment on the effect of measurement errors on the final stability estimate. We show that, at most, the measurement errors are amplified by a constant factor that is independent of time and that is inversely proportional to the nudging parameter. In addition, our proofs give some insight on how to choose the nudging parameter of the observer system to balance convergence rate, discrete error and the amplification of measurement errors.

[1] J. Giesselmann, M. Gugat, and T. Kunkel, "Observer-based data assimilation for barotropic gas transport using distributed measurements", *Comm. Math. Sci.*, vol. 22, no. 8, pp. 2271–2309, 2024.

[2] H. Egger, J. Giesselmann, T. Kunkel, and N. Philippi, "An asymptotic-preserving discretization scheme for gas transport in pipe networks", *IMA J. Numer. Anal.*, vol. 43, no. 4, pp. 2137–2168, 2022.

[3] A. Azouani, E. Olson and E. S. Titi, "Continuous data assimilation using general interpolant observables", *J. Nonlinear Sci.*, 24 (2014), 277-304.

[4] A. Chaumet, J. Giesselmann, "Convergence analysis of a fully discrete observer for data assimilation of the barotropic Euler equations", *in preparation*, 2026.

Dissipative weak solutions for the multi-component Flows

Agnihotri, Jaya; Öffner, Philipp

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Tuesday, May 26, 16:50-17:15, 17.92 – 9.019

The well-posedness of weak (entropy) solutions for hyperbolic systems of conservation laws remains a major problem, as solutions often develop discontinuities and fine-scale oscillations. To address this, more general solution concepts like dissipative weak (DW) (measure-valued) solutions have been successfully introduced in case of the Euler and Navier-Stokes equations [1]. Here, we adapt the DW framework and focus on multi-component flows where we see similar behaviour. In this talk, we present a convergence analysis for a class of structure-preserving finite volume schemes applied to the multi-component compressible Euler system. In particular, we consider schemes that preserve key physical invariant domains of the mixture, specifically the positivity of partial densities, mixture pressure, and internal energy. By using a priori stability and consistency estimates derived, we demonstrate the existence via convergence of our FV methods considered in the framework of DW solutions. Finally, utilizing the weak-strong uniqueness principle, we conclude by demonstrating that our approximate solutions converge pointwise to the regular solution of the system throughout its maximal interval of existence.

[1] E. Feireisl, M. Lukáčová-Medviďová, H. Mizerová, and B. She, *Numerical Analysis of Compressible Fluid Flows*, Vol. 20, Springer, 2021.

Numerical study of dissipative measure-valued solutions for the compressible Euler system

Changsheng Yu

Johannes Gutenberg-Universität Mainz, Germany

Tuesday, May 26, 17:15-17:40, 17.92 – 9.019

This work investigates the numerical approximation for the compressible Euler system with a focus on the Kelvin-Helmholtz instability, where small initial perturbations yield complex vortex structures evolving in time.

We adopt the framework of dissipative measure-valued (DMV) solutions to characterize the limit of consistent and stable numerical approximations as the mesh size vanishes. These DMV solutions are approximated via $\mathcal{K}$-convergence. Our numerical simulations indicate that different numerical schemes can converge to different DMV solutions, demonstrating the inherent non-uniqueness of measure-valued solutions obtained via numerical approximations. Motivated by this fact, we investigate additional admissible criteria to select the physically-relevant solution. We firstly focus on Dafermos' entropy rate criterion, which prioritizes solutions maximizing the local rate of entropy production. Our main theoretical result proves that any solution selected by this criterion from the set of DMV solutions must necessarily be a weak solution. However, our numerical findings present a striking contrast: the approximations exhibit oscillations, indicating that the sequence converges only weakly, which implies that its limit cannot be a weak solution.

We further analyze different higher-order structure-preserving numerical methods with respect to maximizing the total entropy and minimizing the mean energy. Numerical experiments indicate that the methods with larger amount of numerical diffusion are prioritized by such selection criteria.

This is a joint work with Shaoshuai Chu (Aachen), Eduard Feireisl (Prague), Michael Herty (Aachen), Alexander Kurganov (SUSTECH), and Lukáčová-Medviďová, Mária (Mainz). It has been supported by the German Research Foundation (DFG) under the grant SPP 2410, Project number 525853336.

Numerical error estimates for the compressible Navier-Stokes equations with potential temperature transport

Schömer, Andreas; Lukáčová-Medviďová, Mária

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Tuesday, May 26, 17:40-18:05, 17.92 – 9.019

We present a priori error estimates for a mixed finite element-finite volume scheme for the compressible Navier-Stokes equations with potential temperature transport. The numerical solutions are computed on polyhedral domains that approximate a sufficiently smooth bounded domain where the strong solution exists (locally in time). The main ingredients for the proof are the relative energy method and a suitable consistency formulation for the numerical scheme.

Acknowledgements. This project was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) in the framework to the collaborative research center "Multiscale Simulation Methods for Soft Matter Systems" (TRR 146) under Project No. 233630050.

P16-1 Dispersive Waves

Dispersive waves in quasilinear hyperbolic systems on a periodic background

Russo, Giovanni; David Ketcheson

University of Catania, Italy

Tuesday, May 26, 14:00-14:25, 17.52 – 5.019

Many physical models are governed by hyperbolic systems of conservation laws, including the Euler equations of compressible gas dynamics, shallow-water models, and ideal magnetohydrodynamics. Due to their quasilinear structure, genuinely nonlinear waves typically evolve into shock discontinuities even from smooth initial data. These systems admit an additional conservation law associated with the entropy–entropy flux pair, which becomes an inequality after shock formation, reflecting irreversibility.

This picture can change when waves propagate on a periodic background. Numerical and analytical evidence [1, 2] shows that nonlinear waves in several hyperbolic systems may avoid shock formation and instead display dispersive behavior.

We present several systems in which hyperbolic waves exhibit dispersive behavior and derive approximate effective equations that explain these phenomena. In all cases, the effective equations are obtained via asymptotic expansion of the solution in a small parameter representing the ratio between the period of the structure and a typical wavelength.

We first consider a one-dimensional shallow-water system with periodic bathymetry [3], comparing detailed numerical solutions with effective dispersive models at increasing orders of approximation. Traveling-wave solutions of the effective models closely match those emerging from the original system, with agreement improving as higher-order terms are included.

As a second example, we consider the two-dimensional shallow-water system [4], with waves propagating in the x-direction while the bathymetry is periodic in the y-direction. As in the one-dimensional case, there is very good agreement between the detailed numerical solution of the two-dimensional system and the solution of the homogenized equations.

The final example concerns the Euler equations of gas dynamics [5]. The stationary background is a state with constant pressure, zero velocity, and a periodic variation in density. Although we cannot prove that the solution remains smooth, the numerical entropy production is extremely small, suggesting either smooth solutions or the formation of shocks with very small amplitude. We exploit this apparent regularity by adopting simple pseudo-spectral numerical methods based on primitive variables in non-conservative form, which provide solutions indistinguishable from those obtained with more sophisticated shock-capturing WENO schemes [6], at a much lower computational cost.

Open problems and related work in progress will be discussed.

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[2] B. Temple and R. Young. *The nonlinear theory of sound.* arXiv:2305.15623, 2023.

[3] D. I. Ketcheson, L. Lóczi, and G. Russo. *SIAM Journal on Mathematical Modeling and Simulation*, 23(1):397–430, 2025.

[4] D. I. Ketcheson and G. Russo. *Journal of Nonlinear Waves*, 1:e1, 2025.

[5] D. I. Ketcheson and G. Russo. *SIAM Journal on Mathematical Modeling and Simulation*, 24(1):187–213, 2025.

[6] D. I. Ketcheson, M. Parsani, and R. J. LeVeque. *SIAM Journal on Scientific Computing*, 35:A351–A377, 2013.

Modeling non-propagating modes by dispersive hyperbolic systems

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Tuesday, May 26, 14:25-14:50, 17.52 – 5.019

In this talk, we discuss a particular example of first-order relaxation hyperbolic equations that has the dispersion relation with a frequency band gap [1]. This means that waves with frequencies within the band gap interval cannot propagate through the medium. Such behavior is typical for acoustic metamaterials, which are engineered to exhibit unique wave propagation properties not found in natural materials. From the continuum mechanics stand point, such a material can be seen as a two-scale medium, with the meta structure being the micro-scale and the bulk material being the macro-scale. The presented mathematical model is formulated as a system of first-order hyperbolic partial differential equations with a special structure of algebraic source terms that describe the reversible energy exchange between the micro- and macro-scale. We shall present the dispersion relation analysis alongside with preliminary numerical results that confirm the existence of the band gap.

[1] I. Peshkov and L. Le Marrec.Modeling Phononic Band Gap in Microstructured Solids Using the Riemann-Cartan Geometric Framework, arXiv:2601.05402, 2025.

Dispersive boundary layers for scalar conservation laws in one space dimension and their stability

Spinolo, Laura V.
IMATI-CNR, Pavia

Tuesday, May 26, 14:50-15:15, 17.52 – 5.019

Consider the following dispersive approximation of the initial-boundary value problem for a scalar conservation law in one space dimension, namely

$$\begin{cases} \partial_t u^\varepsilon + \partial_x[f(u^\varepsilon)] + \varepsilon^2 \partial_{xxx} u^\varepsilon = 0, & (t,x) \in \mathbb{R}_+ \times \mathbb{R}_+ \\ u^\varepsilon|_{t=0} = u_{in}, \quad u^\varepsilon|_{x=0} = u_b. \end{cases}$$

In the previous expression, the data u_{in} and u_b, as well as the flux function f, are regular, and the characteristic speed f' is strictly negative. In the vanishing dispersion limit $,\varepsilon \to 0^+$, the transient behavior near the boundary is described by the so-called boundary layers, which are steady solutions of the dispersive equation. In particular, I will discuss how one can rigorously establish the validity of the boundary layer expansion as long as the solution of the underlying scalar conservation law remains smooth (this is the so-called "boundary layer stability"). My presentation will be based on joint work with Paolo Antonelli and Pierangelo Marcati.

Effective dispersive model and resonance in fluids with periodic geometry

Busaleh, Laila; Ketcheson, David

Department of Mathematics, College of Science and Humanities, Jubail, Imam Abdulrahman Bin Faisal University, Saudi Arabia

Tuesday, May 26, 15:15-15:40, 17.52 – 5.019

In this talk, we will focus on analyzing the behavior of a fluid propagating in one-dimensional narrow pipe with periodically-varying cross-sectional area, described by a system of first-order hyperbolic Isentropic gas equations. In general, shocks form in the solutions of first-order nonlinear hyperbolic PDEs with constant coefficients, where solitary waves arise in the solutions of first-order nonlinear hyperbolic PDEs with variable coefficients, those solitary waves occur due to the coupling of nonlinearity and dispersive effects that come from the medium's heterogeneity. We derive homogenized effective equations using multiple-scale perturbation theory, which takes the form of a constant-coefficient system including higher-order dispersive terms. We show that pseudospectral solutions to these equations agree well with direct solutions of the isentropic gas equation in periodically-varying cross-sectional area using a high-resolution finite volume method. In the second part of the talk, we present preliminary numerical results for the shallow water equations showing the existence of periodic, non-breaking solutions and the emergence of resonance in bounded domains under specific geometric and parametric conditions.

P17-1 Dimensionally Reduced Flows

A double coordinate system for submarine landslides and generated tsunamis

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Tuesday, May 26, 16:00-16:25, 17.52 – 5.019

Submarine landslides are geological processes in which the sedimentary instability causes a part of the seafloor to collapse. As the displaced volume can be important, it may generate a tsunami and thus pose a great risk to coastal communities. Example of recent landslide-caused tsunamis include the 1979 Nice and the 2003 Balearic island tsunamis [1].

In depth-averaged models, local coordinates are commonly used to model landslide dynamics. Nevertheless their use in submarine landslide models locates the generated tsunami wave far from the real emplacement. In this regard, cartesian coordinates give a better approach but the landslide layer is not well described, on one hand because the velocity does not follow the bottom direction, and on the other hand because the landslide profile is limited. A new approach has been recently proposed in [2], where local coordinates from a constant slope are used to model the landslide layer and cartesian coordinates to model the fluid layer. Based on this work, we propose a new double coordinate system using curvilinear coordinates for the landslide and cartesian coordinates for the fluid layer. Moreover, a multilayer approach is used to describe this fluid part in order to capture the vertical effects.

We will present the model derivation from Navier-Stokes equations, the layer-averaging procedure and simulation results, as well as the expected advantages of this model.

[1] M. Rodriguez, C. Maleuvre, M. Jollivet-Castelot, E. d'Acremont, A. Rabaute, M. Lafosse, G. Ercilla, J.-T. Vázquez, B. Alonso, A. Ammar, C. Gorini, *Tsunamigenic submarine landslides along the Xauen–Tofiño banks in the Alboran Sea (Western Mediterranean Sea).* In: Geophysical Journal International 209 (2017).

[2] J. M. Delgado-Sánchez, F. Bouchut, E. D. Fernández-Nieto, A. Mangeney, G. Narbona-Reina, *A two-layer shallow flow model with two axes of integration, well-balanced discretization and application to submarine avalanches.* In: Journal of Computational Physics 406 (2020).

A depth-integrated model for dry granular flows in local curvilinear coordinates with friction correction

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Tuesday, May 26, 16:25-16:50, 17.52 – 5.019

Depth-integrated models provide a fundamental tool for the description of gravity-driven flows in which one spatial dimension is much smaller than the others, and they are widely used in geophysical and industrial applications, including dry granular flows and avalanches. When such flows evolve over curved topographies, intrinsic or curvilinear coordinate systems are required to properly account for geometric effects arising from the dimensional reduction of the Navier–Stokes equations.

In this work, we propose a new geometric depth-integrated model for dry granular flows defined along a regular reference curve, in which the dependence of the geometric terms on

both the curve parameter and the normal direction is explicitly retained. This leads to a more general formulation than existing intrinsic shallow models and introduces additional conservative variables that play a key role in ensuring mass conservation.

A second major contribution concerns the modelling of basal Coulomb friction in curvilinear coordinates. Extending recent results on corrected bed pressure formulations, we derive a generalized static equilibrium criterion consistent with the local geometry of arbitrary curves, thus avoiding incorrect predictions of the motion threshold. The resulting system can be written as a balance law with source terms and admits physically relevant stationary solutions corresponding to material at rest.

To accurately capture these equilibria, we develop a well-balanced finite volume scheme based on the Polynomial Viscosity Matrix framework combined with a hydrostatic reconstruction adapted to the proposed friction correction. Numerical experiments demonstrate the robustness of the method and highlight the importance of the geometric formulation and the corrected friction criterion for flows over curved geometries.

A new model for two-layer liquid-gas stratified flows in pipes with general cross sections

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National Autonomous University of Mexico, México

Tuesday, May 26, 16:50-17:15, 17.52 – 5.019

In this work, we derive a new model for immiscible two-layer gas-liquid stratified flows in pipes with arbitrary geometries. The two layers are separated by an interface at which streamline continuity conditions are imposed. The bottom layer consists of an incompressible liquid governed by hydrostatic pressure, following a shallow water-type approximation. The upper layer is occupied by a compressible gas modeled by an ideal gas law, leading to conservation equations for mass, momentum, and energy.

The model is obtained through cross-sectional averaging. By integrating the three-dimensional equations of motion over each cross section and applying Reynolds' transport theorem, we derive evolution equations for the cross-sectional averages of the conserved variables. The averaging is performed separately in each layer, resulting in two coupled subsystems of balance laws. The subsystem associated with the lower layer resembles the shallow water equations for an incompressible fluid with hydrostatic pressure, while the upper layer is governed by equations analogous to the compressible Euler system. The two subsystems are coupled through non-conservative products representing momentum and energy exchanges across the interface.

The resulting coupled system consists of five equations. The first two correspond to mass conservation and momentum balance in the liquid layer, governed by constant density and hydrostatic pressure. The remaining three equations describe mass conservation, momentum balance, and energy conservation in the gas layer, with pressure determined by an equation of state relating internal energy and density. The system is conditionally hyperbolic and admits an entropy pair together with an associated entropy inequality. In the regime of small density ratios between layers, we derive approximations for the eigenvalues of the system, which allow us to identify two genuinely nonlinear characteristic fields associated with the shallow water subsystem. The remaining three eigenvalues correspond to the Euler subsystem, consisting of one linearly degenerate field (a contact discontinuity) and two genuinely nonlinear fields.

Numerical experiments are presented to demonstrate the performance of the model and the proposed numerical approximations. These include tests of well-balancedness, Riemann problems, and the convergence of perturbations toward steady states at rest. The results show that

the model accurately captures different flow regimes. In particular, when the density difference between layers is large (e.g., water and air), perturbations in the gas layer induce negligible momentum and energy exchange, leading to predominantly one-way coupling in which the liquid strongly influences the gas dynamics. Conversely, when the density difference is moderate, momentum and energy exchanges become significant in both directions. The numerical results confirm these behaviors, along with other key properties of the model.

Hyperbolic modelling for the thin film flows

Barthwal, Rahul

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Tuesday, May 26, 17:15-17:40, 17.52 – 5.019

Joint work with **Rohde, Christian** (University of Stuttgart, Stuttgart, Germany), **Dhaouadi, Firas** (INRIA Bordeaux, France) and **Wang, Yue** (IAPCM, Beijing, P.R. China)

We introduce hyperbolic models arising in the mathematical study of thin film flows, encompassing both reduced first-order lubrication models and hyperbolic relaxations of fourth-order thin-film equations.

First, starting from a fourth-order two-layer thin film model with a perfectly soluble solute, we derive a reduced one-dimensional hyperbolic system governing the leading-order dynamics of the flow [1]. This system is a rich hyperbolic system, as it admits an entire class of entropy functionals and can be decoupled along its Riemann invariants [1]. Building on a first-order Godunov scheme, we construct a temporally and spatially coupled second-order generalized Riemann problem-based finite-volume method for this system. Numerical experiments, including systematic comparisons with MUSCL finite-volume schemes combined with Runge–Kutta time stepping, demonstrate the accuracy, efficiency, and robustness of the higher-order approach [2].

To address the full fourth-order nonlinear terms arising in scalar thin film equations, we propose a novel hyperbolic relaxation system. Our relaxation formulation is designed to accommodate a broad family of mobility and pressure functions, therefore covering a wide range of thin-film and Cahn–Hilliard-type equations. We derive an explicit energy structure of the proposed system, which converges to the energy (Lyapunov) functional of the original equation when relaxation parameters vanish. This, in particular, demonstrates that the relaxation system preserves the key structural properties of the underlying model. Relying on the relative entropy framework [3], we then establish the convergence of the weak entropy solutions of the proposed relaxation system to the smooth solutions of the original equation in arbitrary space dimensions.
Together, these results provide a coherent hyperbolic viewpoint on the complex thin film dynamics, including both reduced first-order models and structure-preserving hyperbolic relaxations of fourth-order equations.

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Conservative discretization of a nonlinear hyperbolic phase-transport system with curvature

Sambandam, Baskar; Gowda, Veerappa

Indian Institute of Technology, Bombay, India

Tuesday, May 26, 17:40-18:05, 17.52 – 5.019

Direct numerical simulation of high-frequency wave propagation in nonlinear media poses significant challenges due to multiscale oscillations and nonlinear focusing effects. Nonlinear geometric optics provides an asymptotically reduced model that describes wave propagation through a coupled system that comprises a phase equation, which is a fully nonlinear Hamilton-Jacobi type equation, and a transport-balance law for wave energy.

Our aim in this work is to develop an efficient numerical method for the nonlinear geometric optics system based on a monotone finite difference discretization of the phase equation and a conservative finite volume formulation of the energy transport equation. The proposed scheme is conservative, stable, and capable of handling nonsmooth solutions, caustics, and nonlinear source terms. The scheme naturally accommodates amplitude-dependent wave speed and background inhomogenities, which is a novel part of this work.

Our numerical experiments demonstrate that the method accurately captures nonlinear ray dynamics, shock formation along rays, and energy concentration phenomena at a fraction of the cost of direct wave solvers. This approach provides a practical computational tool for high-frequency wave propagation in acoustics and related applications.

P18-1 Scientific Computing

Adaptive schemes for hyperbolic systems of conservation laws

Kurganov, Alexander

Southern University of Science and Technology (SUSTech), China

Thursday, May 28, 11:10-11:35, M17.01

I will present new adaptive schemes for the one- and two-dimensional hyperbolic systems of conservation laws. Our schemes are based on an adaption strategy recently introduced in [1]. We use a smoothness indicator to automatically detect "rough" parts of the computed solution and within the "rough" areas, we detect neighborhoods of linearly degenerate contact discontinuities, in which we employ the second-order finite-volume low-dissipation central-upwind scheme with an overcompressive limiter, which helps to sharply resolve contact discontinuities. In the rest of the "rough" areas, we switch to the dissipative Minmod2 limiter, which is sufficiently sharp to accurately resolve genuinely nonlinear shock waves. Finally, in smooth parts, we replace the limited second-order scheme with a quasi-linear high-order (either fifth, seventh, or ninth order in space and third order in time) finite-difference scheme, recently proposed in [2, 3]. I will demonstrate the advantage of the resulting adaptive schemes on a number of challenging numerical examples for both one- and two-dimensional Euler equations of gas dynamics.

[1] S. Chu, A. Kurganov, and I. Menshov. New adaptive low-dissipation central-upwind schemes. *Applied Numerical Mathematics*, 209:155–170, 2025.

[2] V. A. Kolotilov, V. V. Ostapenko, and N. A. Khandeeva. Finite-difference schemes of odd order in space with increased accuracy in the areas of shock waves influence. Submitted.

[3] V. A. Kolotilov, V. V. Ostapenko, and N. A. Khandeeva. Fifth-order finite-difference scheme in space with increased accuracy in shock influence areas. *Computational Mathematics and Mathematical Physics*, 65:901–916, 2025.

Quantum computation of Hamilton-Jacobi equations: multivalued and viscosity solutions, and quantum scientific computing platform "UnitaryLab"

Jin, Shi

Shanghai Jiao Tong University, China, People's Republic of

Thursday, May 28, 11:35-12:00, M17.01

Quantum computers are designed based on quantum mechanics principle, they are most suitable to solve the Schrodinger equation, and linear PDEs (and ODEs) evolved by unitary operators. Nonlinear PDEs are difficult due to the no-cloning theorem.

In this talk we will show how to construct quantum algorithms for (nonlinear) Hamiton-Jacobi equations. Such equations develop singularities (caustics) in finite time even if the initial data are smooth. Beyond the caustics two possible solutions could be introduced: multivalued solutions which arise in geometric optics, high frequence or semiclassical limits of linear waves, etc, and viscosity solutions which arise in optimal control, level set methods for front propagations, etc. For multivalued solution our quantum algorithm is based on the level set method, while for viscosity solution it is based on an entropy penalization method.

We will also introduce a quantum scientific computing platform—"UnitaryLab", which is a software for quantum algorithms for scientific computing problems. The version 1.0 aims at solving ordinary and partial differential equations by quantum algorithms.

P19-1 Hyperbolic Modelling

Localization phenomena in chemotaxis and in inelastic fracture

Tzavaras, Athanasios

Applied Mathematics and Computational Science, KAUST, Saudi Arabia

Thursday, May 28, 11:10-11:35, M17.02

The goal of this talk is to present three examples that appear in diverse areas of applications and share the mathematical feature of exhibiting localization phenomena, that is phenomena where a variable develops a concentration along the evolution. They are:

- High speed plastic deformations of viscoplastic rate-softening material modeled by a hyperbolic-parabolic system with strain dependent viscosity
- A porous-media type equation with viscosity controlled by a relaxation equation appearing in chemotaxis
- Initiation of fracture in inelastic materials with strain-softening

Through these a theory of localization will be outlined.

On the equivalence of some non-conservative hyperbolic systems and conservative systems

Warnecke, Gerald; Thein, Ferdinand

Otto von Guericke University, Germany

Thursday, May 28, 11:35-12:00, M17.02

Systems of equations are considered to be equivalent if they have the same solution set. For conservation laws there are two types of equivalences. Systems may be equivalent for everywhere differentiable solutions. These we call smoothly equivalent. They may not be equivalent for weak solutions. Systems are called fully equivalent if they additionally have equivalent jump relations for weak solutions. Various examples and some implications for weak solutions of non-conservative systems are discussed.

P20-1 Mixed-Type Problems

Shock wavefronts for forward-backward-forward parabolic equations

Corli, Andrea
University of Ferrara, Italy

Thursday, May 28, 14:00-14:25, M17.23 – 2.011

Cardenas We consider the equation

$$u_t = P(u)_{xx} + g(u), \qquad t \geq 0,\, x \in \mathbb{R}.$$

The unknown function u is assumed to have range in $[0,1]$ and $g(0) = g(1) = 0$; we are interested in *discontinuous* traveling-wave solutions $u(x,t) = \varphi(x - ct)$, for some profile φ and propagation speed $c \in \mathbb{R}$ connecting 0 to 1 (*shock wavefronts*); assume for instance $\varphi(-\infty) = 1$ and $\varphi(\infty) = 0$. In the main framework we assume that the diffusive potential P is not monotone and g changes sign. For definiteness we focus on the case when there exist real numbers satisfying

$$0 < \alpha < \gamma < \beta < 1$$

and assume

(P) $P \in C^2[0,1]$, $P' > 0$ in $(0,\alpha) \cup (\beta,1)$ and $P' < 0$ in (α,β);

(g) $g \in C^0[0,1]$, $g(0) = g(1) = 0$ and $g < 0$ in $(0,\gamma)$, $g > 0$ in $(\gamma,1)$.

Then the equation is *forward-backward-forward* parabolic; a source term g satisfying (g) is called *bistable*. It is known that in this case the equation has *no* regular wavefront. Our main results are the following.

1. If $P(1) > P(0)$ there exists an interval $\mathcal{I} \subset [0,\alpha]$ such that, for every $\varphi_\ell \in \mathcal{I}$, there are unique $\varphi_r \in [\beta,1]$ and $c^* \in \mathbb{R}$ for which there exists a shock wavefront with speed c^* whose profile φ has a single jump from φ_r to φ_ℓ. Moreover, φ_r is determined by $P(\varphi_r) = P(\varphi_\ell)$. There are no other shock wavefronts.
2. If $P(1) = P(0)$ there are piecewise-constant shock-wavefronts whose profile jumps from 1 to 0 and has speed $c = 0$. If $P(1) = P(\gamma) = P(0)$, then there are also piecewise-constant shock-wavefronts whose profiles have two jumps, 1 to γ and γ to 0; the speed is again 0. There are no other shock wavefronts.
3. If $P(1) < P(0)$ there are no shock wavefronts.

Notice that the condition $P(\varphi_r) = P(\varphi_\ell)$ is an equal-area condition for the diffusivity, as for the flux in several hyperbolic phase-transition models. The speed c^* is determined by a Rankine-Hugoniot type condition involving the diffusive flux. The techniques to face the problem are a singular order reduction and the pasting of different stretches of profiles.

This is a joint paper with Diego Berti (University of Turin, Italy) and Luisa Malaguti (University of Modena-Reggio Emilia, Italy).

Many species system with nonlocal interactions driven by Riesz potentials

Iorio, Valeria

University of L'Aquila, Italy

Thursday, May 28, 14:25-14:50, M17.23 – 2.011

In this talk, we consider a many species system with repulsive nonlocal velocity fields driven by fractional Laplacian operators. We establish the existence of weak measure solutions for initial data in the space of probability measures with finite second moment and finite fractional Sobolev norms. In the proofs we adopt the Jordan–Kinderlehrer–Otto scheme combined with the flow interchange technique.

Localization of self-similar solutions in diffusion–relaxation systems

Kim, Hoyoun

King Abdullah University of Science and Technology (KAUST), Kingdom of Saudi Arabia

Thursday, May 28, 14:50-15:15, M17.23 – 2.011

In this talk, we construct localizing solutions for diffusion–relaxation systems by exploiting their self-similar structure. Diffusion–relaxation models arise in a wide range of applications, from shear-induced motion to biological chemotaxis, and exhibit equilibration and localization depending on parameter regimes. We extend the self-similar formulation to multi-dimensional domains and analyze the resulting asymptotic behavior of solutions. Using geometric singular perturbation theory and the Poincaré–Bendixson lemma on a perturbed invariant manifold, we establish the existence of localized self-similar solutions.

P20-2 Mixed-Type Problems

The Hele-Shaw free boundary limit of Buckley-Leverett system

Gomes, André; Neves Wladimir

Federal University of Rio de Janeiro, Brazil

Thursday, May 28, 16:00-16:25, M17.23 – 2.011

We propose a new approach to solving the Buckley-Leverett System, which is to consider a compressible approximation model characterized by a stiff pressure law. Passing to the incompressible limit, the compressible model gives rise to a Hele-Shaw type free boundary limit of Buckley-Leverett System, and it is shown the existence of a weak solution of it.

A bulk-surface Cahn-Hilliard model for chemically active wetting

Kamath Katapady, Megha; Fagioli, Simone; Pietschmann, Jan-Frederik

University of L'Aquila, Italy

Thursday, May 28, 16:25-16:50, M17.23 – 2.011

In this talk, we present a bulk-surface coupled Cahn-Hilliard system with an active source term that models the wetting of a solute-solvent mixture (in the bulk) on an active membrane (surface). The active binding process at the surface maintains the system away from equilibrium leading to the formation of non-conventional stationary droplet shapes, as seen in numerical simulations. We aim to show the existence of weak solutions to the system that involves non-linear surface interactions and degenerate mobility terms. We start with a non-degenerate version of the model on which we perform a time-discretization and show the existence of iterates via a Babuška-Lax-Milgram argument. Further, some discrete energy estimates allow us to pass to the continuum limit. We then argue for additional regularity estimates using appropriate test functions that allow us to pass to the degenerate limit. This is an ongoing work with S. Fagioli and J-F. Pietschmann.

String stability for hysteretic flows

Fan, Haitao; Corli, Andrea

Georgetown University Washington DC, United States of America

Thursday, May 28, 16:50-17:15, M17.23 – 2.011

We investigate string stability in continuum flow models where information propagates to the left. String stability concerns whether oscillations decay or grow as they travel, with rates measured in the spatial variable. In traffic flow, this corresponds to how speed oscillations of successive vehicles evolve as disturbances move upstream.

Positive hysteresis loops my amplify flux oscillations and can generate shocks. For hysteretic LWR model, a solution containing a large enough hysteresis loop in the positive hysteresis region will develop stop-and-go waves, else the flux oscillations will decay as they propagate upstream. A necessary and sufficient condition for this "large enough" is found. In the region of negative hysteresis, hysteresis loops damp flux oscillations. For this case, we formally derive spatial decay rates for time-periodic string-stability solutions for several hysteretic models such as hysteretic LWR model, hysteretic ARZ model, transport with absorption and desorption model. Similar decay rates is also obtained for spatially periodic solutions of Amadori-Bressan-Shen hysteretic flow model.

P21-1 Relaxation Approximation

Relaxation schemes for coupled systems of conservation laws

Kolbe, Niklas

RWTH Aachen University, Germany

Thursday, May 28, 14:00-14:25, M17.21 – 2.005

We consider coupled systems of conservation laws posed in different spatial subdomains of a common computational domain that are linked through interface coupling conditions.Those arise for instance in composite materials, multiphase flows, and multi-dimensional fluid–structure interaction. In this context, we introduce a new class of relaxation-based Runge–Kutta discontinuous Galerkin schemes for multi-dimensional coupled hyperbolic problems separated by fixed sharp interfaces. The schemes are derived employing Jin–Xin relaxation combined with a local projection and are constructed not to require the expensive solution of nonlinear half-Riemann problems. This leads to a black-box-like approach for the user that can be applied in multiple space dimensions without problem-specific flux decomposition at the interfaces.

To enforce the coupling conditions at the interface on the relaxation level, a problem-adapted modification of the coupling condition is introduced. A simple and general algorithm is proposed that guides this construction for a wide class of coupled hyperbolic systems, and the resulting schemes are shown to preserve simple imposed coupling rules, such as Kirchhoff-type conditions, on the discrete level. Higher order accuracy in time is obtained via strong stability preserving Runge–Kutta methods, derived from an asymptotic preserving implicit–explicit treatment of the coupled relaxation system in the discrete relaxation limit. In a multi-dimensional case study, we demonstrate the applicability and robustness of the approach for fluid–structure coupling problems involving the compressible Euler equations and a linear elastic solid model.

Hyperbolic relaxation of the Navier–Stokes–Cahn–Hilliard system for incompressible two-phase flows

Konan, Hasel-Cicek; Rohde, Christian

University of Stuttgart, Germany

Thursday, May 28, 14:25-14:50, M17.21 – 2.005

Diffuse-interface models have gained increasing attention for the simulation of two-phase flows. They describe topological changes between immiscible or partially miscible fluids. In this work, we consider a diffuse-interface model for two incompressible and immiscible fluids at a constant temperature. To investigate the interface dynamics of such flows, we focus on the incompressible Navier–Stokes–Cahn–Hilliard (NSCH) system, which is thermodynamically consistent. Despite its advantages, the computational cost of the NSCH system is high due to the presence of nonlocal nonlinear terms, the fourth-order Cahn–Hilliard equation, and the pressure Poisson problem. To address these challenges, we approximate the inviscid part of the NSCH system by a first-order hyperbolic relaxation system. In addition to preserving thermodynamic consistency, this approximation enables the use of classical numerical methods for hyperbolic balance laws and reduces computational cost by avoiding the discretization of the fourth-order derivative and the solution of the pressure Poisson equation. A characteristic analysis in one spatial dimension yields asymptotic expressions for four real and distinct eigenvalues, which are derived up to a prescribed asymptotic order. As a first validation, numerical experiments are performed in one spatial dimension using a classical second-order finite-volume MUSCL–Hancock scheme with a Rusanov numerical flux. The numerical results accurately capture the wave structure of the

hyperbolic system. Moreover, we consider three benchmark test cases: a stationary droplet, spinodal decomposition, and Ostwald ripening. The results demonstrate the convergence of the approximate solutions toward the original NSCH solution as the relaxation parameters tend to zero. We further show that the proposed numerical schemes are energy dissipative for all test cases. In addition, a comparison of computational costs for the stationary droplet benchmark confirms the efficiency of the hyperbolic approximation. Finally, we develop an asymptotic preserving scheme for the relaxation system that remains uniformly stable with respect to the relaxation parameters and recovers the original NSCH dynamics in the vanishing relaxation limit without resolving the small relaxation scales.

A hyperbolic approximation of the nonlinear Schrödinger equation

C. Muñoz-Moncayo; A. Biswas; L. S. Busaleh; D. I. Ketcheson; M. Rajvanshi

King Abdullah University of Science and Technology (KAUST)

Thursday, May 28, 14:50-15:15, M17.21 – 2.005 We study a first-order hyperbolic approximation of the nonlinear Schrödinger (NLS) equation in one and higher spatial dimensions. The proposed hyperbolic system is shown to be strictly hyperbolic and to possess a modified Hamiltonian structure, along with conserved quantities that approximate the mass and energy of NLS. We establish the existence of standing-wave solutions (explicit in the one-dimensional case) that are closely related to ground-state solutions of NLS. Moreover, we show that solutions of the hyperbolic system converge to those of NLS in the relaxation limit under appropriate regularity assumptions.

We develop high-order asymptotic-preserving numerical discretizations that converge to consistent discretizations of NLS while exactly conserving mass. Furthermore, a robust and fully explicit finite volume scheme for the hyperbolic system is proposed. This discretization allows us to use patch-based adaptive mesh refinement on logically rectangular mapped grids with non-periodic boundary conditions. Numerical experiments in both the focusing and defocusing regimes demonstrate the accuracy and robustness of the approach, with applications including Bose–Einstein condensates, line-soliton instabilities, and superflow past obstacles.

[1] A. Biswas, L. S. Busaleh, D. I. Ketcheson, C. Muñoz-Moncayo and M. Rajvanshi, "A Hyperbolic Approximation of the Nonlinear Schrödinger Equation", *Studies in Applied Mathematics*, 155, 2025.

Relative entropy method for relaxation of fluid systems

Mahmoud, Christina; Mathis, Hélène; Seguin, Nicolas

Institut Montpelliérain Alexandre Grothendieck (IMAG), Université de Montpellier, CNRS; Inria, Team ANGUS, Montpellier, France

Thursday, May 28, 15:15-15:40, M17.21 – 2.005

Multiphase compressible flow modeling relies on a broad hierarchy of hyperbolic balance laws coupled with stiff relaxation. Motivated by classical two-phase models [1, 6], we study hyperbolic relaxation systems in Lagrangian coordinates [4]. Our goal is to exploit the dissipation due to relaxation and the entropy structure of the nonlinear hyperbolic system to get quantitative stability estimates within the relative entropy approach [3, 7, 8].

We introduce a relative entropy functional that quantifies the deviation from a smooth reference solution, typically associated with the equilibrium model, and we derive a local relative entropy identity. The stiff source term produces an explicit dissipative term controlling the deviation in the relaxed variables, while the remaining contributions are estimated through linearization remainders and the regularity of the reference state. This yields quantitative stability bounds and a rigorous justification of the stiff relaxation limit as the relaxation time tends to zero, as long as the reference solution remains smooth. The framework connects to relaxation models for phase-transition flows [2, 5] and more generally to two-phase models with physically motivated relaxation mechanisms.

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[7] A. Tzavaras. Relative entropy in hyperbolic relaxation. *Communications in Mathematical Sciences*, 3(2):119–132, 2005.

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P21-2 Relaxation Approximation

On hyperbolic approximations for a class of dispersive and diffusive-dispersive equations

Dhaouadi, Firas; Barthwal, Rahul; Rohde, Christian

Bordeaux INP, Institut de Mathématiques de Bordeaux, Université de Bordeaux, CNRS UMR 5251, INRIA, F-33400, Talence, France

Thursday, May 28, 16:00-16:25, M17.21 – 2.005

We introduce new approximate systems for dispersive and diffusive-dispersive equations with nonlinear fluxes. For purely dispersive equations, we construct a first-order, strictly hyperbolic approximation. Local well-posedness of smooth solutions is achieved by constructing a unique symmetrizer that applies to arbitrary smooth fluxes. Under stronger conditions on the fluxes, we provide a strictly convex entropy for the hyperbolic system that corresponds to the energy of the underlying dispersive equation. To approximate diffusive-dispersive equations, we rely on a viscoelastic damped system that is compatible with the found entropy for the hyperbolic approximation of the dispersive evolution. For the resulting hyperbolic-parabolic approximation, we provide a global well-posedness result. The structure of the new approximate systems allows to apply standard numerical simulation methods from the field of hyperbolic balance laws. We confirm the convergence of our approximations even beyond the validity range of our theoretical findings on a set of test cases covering different target equations. We show the applicability of the approach for strong nonlinear effects leading to oscillating or shock-layer-forming behavior.

A hyperbolic relaxation approximation of the incompressible Navier-Stokes equations: analysis and numerical results

Huang, Qian; Rohde, Christian; Yong, Wen-An; Zhang, Ruixi

University of Stuttgart, Germany

Thursday, May 28, 16:25-16:50, M17.21 – 2.005

Consider the incompressible Navier-Stokes equations

$$\begin{cases} \nabla \cdot u = 0, \\ \partial_t u + \nabla \cdot (u \otimes u) = -\nabla p + \Delta u, \\ u(0, \cdot) = u_0 \end{cases} \tag{21}$$

for $(t, x) \in [0, T_0] \times \mathbb{T}^d$. Require that $\int_{\mathbb{T}^d} p(t, x)\, dx = 0$ for all t. By introducing $U = u \otimes u - \nabla u \in \mathbb{R}^{d \times d}$, the momentum equation can be rewritten as

$$\partial_t u + \nabla \cdot U + \nabla p = 0.$$

We introduce a novel first-order hyperbolic relaxation approximation to (21) as follows:

$$\begin{cases} \epsilon \partial_t p^{\epsilon,\delta} + \nabla \cdot u^{\epsilon,\delta} = 0, \\ \partial_t u^{\epsilon,\delta} + \nabla \cdot U^{\epsilon,\delta} + \nabla p^{\epsilon,\delta} = 0, \\ \delta \partial_t U^{\epsilon,\delta} + \nabla u^{\epsilon,\delta} = u^{\epsilon,\delta} \otimes u^{\epsilon,\delta} - U^{\epsilon,\delta}, \\ u^{\epsilon,\delta}(0, \cdot) = u_0^{\epsilon,\delta},\ p^{\epsilon,\delta}(0, \cdot) = p_0^{\epsilon,\delta},\ U^{\epsilon,\delta}(0, \cdot) = U_0^{\epsilon,\delta}, \end{cases}$$

where $\epsilon, \delta > 0$ are two small parameters. With vanishingly small perturbations of initial velocity, we rigorously prove the simultaneous convergence of fluid velocity and pressure toward the Navier-Stokes limit in both two- and three-dimensional cases by constructing an intermediate affine

system to obtain the necessary error estimates for the pressure. Furthermore, we extend the velocity convergence analysis to the case of $\mathcal{O}(1)$ initial velocity perturbations, and establish the global-in-time recovery of the velocity field using a modulated energy structure and delicate bootstrap arguments in both two- and three-dimensional settings. A series of numerical tests are performed to demonstrate the effectiveness of the proposed hyperbolic relaxation approximation.

A structure preserving scheme for the Alber equation, and investigation of the onset of instability

Athanassoulis, Agissilaos

University of Dundee, United Kingdom

Thursday, May 28, 16:50-17:15, M17.21 – 2.005

The Alber equation [1] has been widely used as a model for stochastic ocean waves, and particularly in connection with the emergence of extreme events, cf., e.g. [6, 4]. This is often based on its linear stability analysis [3], describing a bifurcation between a stable regime akin to Landau damping, and a modulationally unstable regime.

In this work explore numerically the onset of instability for the fully nonlinear problem. To that end, first we analyze the impact of periodizing the problem on a finite computational domain to the stability condition [2]. We proceed to propose a structure-preserving, second-order-in-time numerical scheme for its numerical approximation, inspired by [5]. The space discretization is carried out with fourth-order finite differences, leading to sparse matrices. The problem's main balance law and two invariants are preserved at the discrete level.

Using this numerical scheme, we explore for the first time nonlinear aspects of the bifurcation. We find a threshold for the instability, below which the instability is present but does not lead to $O(1)$ effects in the sea state. This resolves long-standing questions in the field.

Extensive Monte Carlo simulation of many initial inhomogeneities is carried out at the HPC cluster of the University of St Andrews. It is found that the behavior described above is not sensitive to the initial inhomogeneity, and depends only on the background spectrum.

This is joint work with Irene Kyza (St Andrews) and Fotini Karakatsani (Ioannina).

[1] Alber, I.E., 1978. The effects of randomness on the stability of two-dimensional surface wavetrains. Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences, 363(1715), pp.525-546.

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Well-balanced kinetic relaxation schemes for hyperbolic systems of balance laws

León Miguel Ávila León; Manuel Jesús Castro Díaz, Victor Michel-Dansac, Emmanuel Franck

University of Málaga, Spain

Thursday, May 28, 17:15-17:40, M17.21 – 2.005

This work presents a novel family of **well-balanced** numerical schemes for hyperbolic systems of balance laws based on the **kinetic relaxation** approach [1]. The method begins by transforming the original non-linear system into a linearized kinetic system with an increased number of variables. In this framework, non-linearities are shifted to the source term, and the connection to the macroscopic variables is maintained via a projection operator related to the Maxwellian equilibrium states.

These relaxed systems are typically solved using **splitting** techniques [3], where the evolution is decomposed into two distinct steps: first, the **transport stage** solving the source-free kinetic system, and second, the **relaxation stage** governed by an ordinary differential equation where the time derivative is balanced by the kinetic source term.

The primary objective is to design schemes that accurately preserve the steady-state solutions of the original macroscopic equations, noting that these do not necessarily coincide with the trivial equilibria of the relaxed system. This **well-balanced** property [2] is achieved through an appropriate **formulation of the kinetic system** combined with **consistent reconstruction operators**. This approach ensures that the balance is maintained throughout both the transport stage and the projection stage, where the treatment of the macroscopic source term is carefully integrated. We demonstrate that this framework can be implemented using both **Finite Volume (FV)** and **Semi-Lagrangian (SL)** techniques in the transport step.

We develop and analyze these schemes for the Finite Volume case in both **explicit** and **implicit** formulations. **High-order accuracy in space** (first, second, and third-order) is attained by employing high-order reconstruction operators, while **higher-order temporal accuracy** is achieved by correctly combining lower-order splitting steps. Furthermore, the Semi-Lagrangian implementation is explored for its stability advantages.

Finally, the performance and robustness of the proposed methods are validated through several numerical benchmarks, including the **Burgers equation with a source term**, **1D Shallow Water Equations (SWE)** with complex bathymetry preserving non-trivial "moving water" equilibria and the **Euler equations under external potentials**.

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P22-1 ALE Methods

An arbitrary Lagrangian-Eulerian semi-implicit FV/FE method for continuum mechanics

Busto, Saray

Galician Center for Mathematical Research and Technology, CITMAga; Universidade de Santiago de Compostela, Spain

Thursday, May 28, 14:00-14:25, M17.25 – 2.017

A novel semi-implicit arbitrary Lagrangian-Eulerian (ALE) method is proposed for the solution of the unified Godunov-Peshkov-Romenski (GPR) model of continuum mechanics which, at the aid of suitable relaxation source terms, is able to describe continuum mechanics problems going from non-linear elasto-plastic solids to viscous fluids. An operator splitting strategy is applied to decouple the computation of the velocity and the fast pressure waves yielding a numerical scheme able to address all Mach number flows. The resulting transport subsystem is solved at the aid of an explicit finite volume method, in which integration over closed space-time control volumes ensures verification of the geometric conservation law (GCL). Meanwhile, implicit continuous finite elements are employed to discretise the Poisson-type subsystem related to the pressure. The potentially stiff source terms, of the distortion and thermal impulse fields, are also advanced implicitly and an asymptotic preserving formulation on the fluid limit of the equations is employed to properly address wall boundary conditions. The properties of the overall method are carefully assessed both in the fluid and solid limits of the model.

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SCOUT: Semi-Lagrangian COnservative and Unconditionally sTable schemes for nonlinear advection-diffusion problems

Preda, Silvia; Boscheri, Walter; Semplice, Matteo; Tavelli, Maurizio

University of Insubria, Italy

Thursday, May 28, 16:25-16:50, M17.25 – 2.017

In this talk, we present a new conservative semi-Lagrangian finite difference scheme for nonlinear advection-diffusion problems. Conservation, which is fundamental for obtaining physically consistent solutions, is ensured by integrating the governing equations over a space-time control volume constructed along the characteristic curves originating from each computational point. The subsequent application of Gauss theorem allows the evaluation of all space-time surface integrals and enables the computation of the time integrals of the flux using only spatial information, which is available at the beginning of the timestep. For nonlinear problems, a nonlinear equation must be solved to determine the foot of the characteristic, whereas this step is not needed in linear cases. Furthermore, we show how the diffusion terms can be directly incorporated within the same framework, leading to the development of a novel characteristic-based Crank-Nicolson discretization in which the diffusion contribution is integrated along the characteristic. A broad set of benchmark tests is presented to assess the accuracy, robustness, and strict conservation property of the proposed method, as well as its unconditional stability, which is verified by numerical experiments with CFL numbers up to 100.

[1] W. Boscheri, M. Tavelli and L. Pareschi. On the construction of conservative semi-Lagrangian IMEX advection schemes for multiscale time dependent PDEs. *J. Sci. Comput.*, 90(3):97, 2022.

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An Arbitrary-Lagrangian-Eulerian solver for relativistic detonation waves

Rinaldi Sara; Zanotti Olindo; Dumbser Michael.

Università di Trento, Italy

Thursday, May 28, 16:50-17:15, M17.25 – 2.017

In this work we present a one dimensional Lagrangian code for relativistic detonations. The problem is described by the equation for the conservation of mass and of energy-momentum, with an extra evolution equation that governs the variation of the reaction rate along the fluid flow. These are analyzed in a one dimensional setting. The shock wave profile is achieved by integrating the PDEs in rest frame of the shock with an Arbitrary Lagrangian Eulerian scheme [2]. This solution is validated with the one obtained though the integration of the associated ODE system, which is reached by performing a Lorentz transformation into the rest frame of the shock front. In the one dimensional setting it is possible to derive the Rankine Hugoniot conditions. These can be used to obtain analytical equations for each variable in relation to the one representing the reaction rate. The ODE system is then reduced to a single ODE for the reaction rate. This ODE is integrated with a well known Runge Kutta method.

[1] M. Dumbser, A. Uuriintsetseg, and O. Zanotti. On arbitrary-lagrangian-eulerian one-step weno schemes for stiff hyperbolic balance laws. *Communications in Computational Physics, 14(2):301 – 327, 2013.*

Simplex solver for Lagrangian hydrodynamics

Xu, Xihua; Xiao, Min

Institute of Applied Physics and Computational Mathematics, Beijing, China

Thursday, May 28, 17:15-17:40, M17.25 – 2.017

Over the past seven decades, Lagrangian staggered-grid hydrodynamics (SGH) and cell-centered hydrodynamics (CCH) have emerged as two dominant paradigms for solving the compressible Euler equations. Although both approaches are formally derived from the same continuum conservation laws, they differ fundamentally in their discretization strategies and numerical behavior: SGH schemes typically rely on artificial viscosity to capture shocks and dissipate kinetic energy, whereas CCH methods employ Riemann solvers at cell interfaces to convert kinetic energy into internal energy. This talk introduces a novel simplex solver that unifies the treatment of

Lagrangian hydrodynamics on both primary and dual meshes. The formulation is rigorously constrained by Newton's three laws of motion, ensuring conservation of mass, momentum, and total energy. Moreover, the discretization satisfies thermodynamic compatibility. The simplex solver requires no empirical parameters, exhibits strong universality, and can be naturally extended to arbitrary 2D polygonal or 3D polyhedral meshes, providing a reliable numerical framework for multiphysics simulations involving complex geometries.

P23-1 Control and Inverse Problems

Existence of spiral strategies for blocking fire spreading

Bianchini, Stefano; Zizza, Martina

SISSA, Trieste, Italy

Thursday, May 28, 14:00-14:25, M17.74 – 7.017

We address the problem for blocking fire by constructing a wall ζ whose shape is spiral-like. This is supposed to be the best strategy when a single firefighter is constructing the wall with a finite construction speed σ: the barriers which satisfy this bound on the construction speed are called admissible.

We prove a sharp version of Bressan's Fire Conjecture in this case, i.e. when admissible barriers are spiral-like curves: namely, there exists a spiral-like barrier confining the fire in a bounded region of $\mathbb{R}^2$ if and only if the speed of construction of the barrier σ is strictly larger than a critical speed $\bar{\sigma} = 2.614....$

The existence of confining spiral barriers for $\sigma > \bar{\sigma}$ is already known, while we concentrate on the negative side, i.e. if $\sigma \leq \bar{\sigma}$ no admissible spiral blocks the fire.

The proof of these results relies on:

1. the precise definition of spiral barrier and its representation;
2. the analysis of saturated spiral barriers as a Retarded Differential Equation (RDE);
3. the equivalent reformulation of the conjecture as a minimum problem of a functional for a prescribed functional;
4. the construction of the optimal closing spiral;
5. the analysis of a differentiable path of admissible spirals along which the functional is differentiable, and in particular increasing when moving from the optimal spiral to any other one (homotopy argument).

Due to the complexity of the solution, the evaluation of the quantities needed to prove that the functional is increasing is performed numerically.

This work is in collaboration with Martina Zizza.

On the optimal control of propagation fronts

Trifone, Chiara; Bianchini, Stefano

SISSA, Italy

Thursday, May 28, 14:25-14:50, M17.74 – 7.017

Following a model proposed by Alberto Bressan and Maria Teresa Chiri ([2], [3]) we study a reaction–diffusion equation describing the evolution of an invasive population. Motivated by a pest–eradication problem, we introduce a control term to be chosen optimally so as to simultaneously minimize the control cost and the population size over time. We focus our analysis on the optimal control of one-dimensional traveling waves and provide an explicit characterization of the unique optimal profile, removing all assumptions previously made in the literature. Moreover, we investigate the regularity of the effort function with respect to the traveling-wave speed. Finally, we explain a possible restatement of the parabolic problem in terms of controlled set evolution and present a Γ-convergence conjecture formulated by Bressan. All results are obtained in collaboration with Stefano Bianchini (SISSA).

[1] S. Bianchini and C. Trifone. "On the properties of traveling wave solutions for a reaction diffusion equation with control". *In Preparation.*

[2] A. Bressan, M.T. Chiri, and N. Salehi. "On the optimal control of propagation fronts". In: *Math. Models Methods Appl. Sci.* 32.06, (2022), pp. 1109–1140.

[3] A. Bressan, M.T. Chiri, and N. Salehi. "Optimal control of moving sets". In: *J. Differential Equation* 361, (2023), pp. 97-137.

Direct and inverse problems in a hypersonic piston model with large velocity variations

Li, Qianfeng

Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany

Thursday, May 28, 14:50-15:15, M17.74 – 7.017

We investigate the direct and inverse problems for a hypersonic piston model describing the expansion of a piston into a rarefied gas. As the initial density tends to zero, the piston approaches the leading shock front, while strict hyperbolicity and shock dissipation degenerate. Through asymptotic analysis, we derive quantitative estimates for these effects and identify a stabilization mechanism. Using characteristic decomposition, we establish global-in-time existence of piecewise smooth solutions for both the direct problem of determining the shock trajectory from the prescribed piston trajectory, and the inverse problem of reconstructing the piston trajectory from a prescribed shock front.

P23-2 Control and Inverse Problems

Galois connections in Hamilton - Jacobi equations and conservation laws

Colombo, Rinaldo M.; Perrollaz, Vincent

University of Brescia, Italy

Thursday, May 28, 16:00-16:25, M17.74 – 7.017

Hamilton - Jacobi equations and conservation laws, namely

$$(\mathbf{HJ}): \begin{cases} \partial_t U + f(x, \partial_x U) = 0 \\ U(0,x) = U_o(x) \end{cases} \qquad \text{and} \qquad (\mathbf{CL}): \begin{cases} \partial_t u + \partial_x f(x,u) = 0 \\ u(0,x) = u_o(x) \end{cases}$$

are well known to be typically non reversible. It is thus natural to consider the following problems.

Reachability: Given $t > 0$ and U_f, respectively u_f, does there exists U_o, respectively u_o, such that the solution to **(HJ)**, respectively **(CL)**, at time t is U_f, respectively u_f?

Inverse Design: Given $t > 0$ and U_f, respectively u_f, characterize its *inverse design*, i.e., the set of those U_o, respectively u_o, such that the solution to **(HJ)**, respectively **(CL)**, at time t is U_f, respectively u_f.

The recent literature [3, 4, 6] tackles these issues with techniques ranging, for instance, from vanishing viscosity approximations to $\mathbf{L^2}$ projections. Where available, the obtained results are not always of immediate interpretation. For instance, why is the inverse design a cone? In which sense is the backward-forward map effective?

Galois Connections offer a natural environment where these problems can be solved, the effective techniques being based on (suitable) *partial orderings* and on the structure of *value functions*, see [5]. As a result, various recent results can be extended, covering, for instance, also time dependent cases, initial boundary value problems and star-shaped networks [1, 2].

Applications of these results are in deep learning and data assimilation, see for instance [7].

Work in collaboration with Vincent Perrollaz, University of Tours.

[1] Adimurthi and S. S. Ghoshal. Exact and optimal controllability for scalar conservation laws with discontinuous flux. *Communications in Contemporary Mathematics*, 25(06):2250024, 2023.

[2] F. Ancona and L. Talamini. Backward-forward characterization of attainable set for conservation laws with spatially discontinuous flux. *Journal de Mathématiques Pures et Appliquées*, page 103814, 2025.

[3] R. M. Colombo and V. Perrollaz. Initial data identification in conservation laws and Hamilton-Jacobi equations. *J. Math. Pures Appl. (9)*, 138:1–27, 2020.

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[5] R. M. Colombo, and V. Perrollaz. Galois Connections in the Inverse Design for Hamilton - Jacobi Equations and Conservation Laws. *In preparation*, 2026.

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Discrete adjoint gradient computation for multiclass traffic flow models on road networks

Mezquita-Nieto, Carmen

RPTU Kaiserslautern, Germany; Inria Université Côte d'Azur, France

Thursday, May 28, 16:25-16:50, M.17.74 – 7.017

This work presents an efficient optimization framework for multi-class traffic flow on road networks using a discrete adjoint gradient approach. We model traffic dynamics as a coupled hyperbolic system of conservation laws, extending the classic Lighthill-Whitham-Richards (LWR) model to accomodate multiple vehicle classes. In this framework, vehicle types such as cars and trucks are distinguished by specific velocity functions that depend on the total traffic density. This approach allows for a more realistic representation of heterogeneous traffic, capturing phenomena such as creeping behavior.

The system of partial differential equations is discretized using a Godunov-type finite volume scheme based on class-specific demand and supply functions. To model realistic traffic networks, we incorporate coupling conditions at junctions. For diverging junctions, we implement a first-in-first-out (FIFO) logic to ensure that congestion on any outgoing branch correctly restricts the flow of all incoming classes. Furthermore, boundary conditions are treated with time-dependent buffer lengths to account for congestion that extends beyond the network boundaries, ensuring that vehicles unable to enter the network during peak periods are preserved in the total mass balance and accounted for into the calculations. The numerical stability of the scheme is guaranteed by adhering to the CFL condition accross all road segments in the network.

The core of this research is the formulation of a PDE-constrained optimization problem designed to minimize travel-related performance metrics, specifically the Total Travel Time (TTT) and Total Travel Distance (TTD). While gradient-based optimization via finite differences is an effective standard approach, the associated computational cost becomes extremely high as the number of time-dependent control variables increases. We address this limitation by relying on the *first-discretize-then-optimize* technique and we formulate the adjoint equations directly on the discretized model. This approach allows us to compute the gradient of the cost functional efficiently, regardless of the number of control parameters. This efficiency is achieved by exploiting the sparse, lower-triangular block structure of the Jacobian matrix of the discretized system.

Numerical simulations conducted on a sample network demonstrate the robustness of the proposed framework. We show a comparative analysis between the adjoint gradient computation and central finite differences. The results show that while the computational cost of the adjoint method does increase with the number of control variables, it does so at a significantly lower pace compared to central finite differences. Furthermore, we explore multi-objective optimization through the generation of Pareto fronts, providing insights into the trade-offs between minimizing travel time and distance. The results indicate that this framework is a useful tool for real-time traffic management and the development of optimal routing strategies in complex, multi-modal transportation networks.

On controllability of conservation laws with space discontinuous flux

Ancona, Fabio; Talamini, Luca

University of Padova, Italy

Thursday, May 28, 16:50-17:15, M.17.74 – 7.017

Consider a scalar conservation law with a spatially discontinuous flux at a single point $x = 0$, and assume that the flux is uniformly convex when $x \neq 0$. I will discuss controllability problems for AB-entropy solutions associated to the so-called (A, B)-interface connection.

I will first present a characterization of the set of profiles of AB-entropy solutions at a time horizon $T > 0$, as fixed points of a backward-forward solution operator.

Next, I will address the problem of identifying the set of initial data driven by the corresponding AB-entropy solution to a given target profile ω^T, at a time horizon $T > 0$.

These results rely on the introduction of proper concepts of AB-backward solution operator, and AB-genuine/interface characteristics associated to an (A, B)-interface connection, and exploit duality properties of backward/forward shocks for AB-entropy solutions.

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A robust optimization framework for sensor networks

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Indian Institute of Technology, Kharagpur, India

Thursday, May 28, 17:15-17:40, M.17.74 – 7.017

Sensor coverage problems arising in environmental monitoring, surveillance, and disaster management can be formulated as constrained feasibility problems under geometric and parametric uncertainty. In directional and aerial sensor networks, small perturbations in sensor position or orientation may lead to severe loss of coverage, motivating the need for mathematically rigorous robustness measures. In this work, we employ tools from robust optimization and convex analysis to introduce the radius of robust feasibility (RRF), which quantifies the maximal admissible uncertainty for which coverage constraints remain satisfied. The RRF is characterized through exact feasibility conditions for directional sensing models and extended to three-dimensional aerial configurations. By exploiting Voronoi-based spatial decompositions, we further develop distributed orientation adjustment algorithms that preserve feasibility under uncertainty. Numerical experiments demonstrate that the proposed framework substantially improves robustness and coverage guarantees compared to deterministic formulations, providing a mathematically grounded approach for resilient sensor network design under uncertainty.

High-fidelity adjoint aerodynamic optimization design based on high-order CFD solvers for NS equations

Wang, Kun; Liu, Tiegang; Zhang, Bin; Wang, Jiaxin

Beihang University, China

Thursday, May 28, 17:40-18:05, M.17.74 – 7.017

Shape optimization based on computational fluid dynamics (CFD) is a crucial approach in aircraft design. As the core of shape optimization, the efficiency, accuracy, and stability of the flow solver significantly impact the optimization process. Nowadays, the second-order finite volume scheme is a workhorse for flow solvers in shape optimization. However, this method suffers from low accuracy and requires sufficiently dense meshes to yield reliable results.
This work focuses on overcoming the computational challenges to enable high-order direct discontinuous Galerkin (DDG) method in large-scale 3D shape optimization. The optimization framework is implemented based on component based HODG CFD framework. In the report, we firstly introduce the latest developments of the HODG framework in heterogeneous parallel computing. To address the insufficient accuracy of 2nd-order flow solvers in current 3D shape optimization, we present the first design and implementation of a 3D shape optimization framework based on the high-order DDG method. To address the high computational cost of high-order DDG method in the shape optimization described above, we propose a lower-bound-preserving solution remapping technique, to accelerate steady-state convergence for intermediate shapes. To address the high computational and storage demands in large-scale 3D shape optimization, we further develop an MPI-based parallel 3D high-order shape optimization framework. Numerical results indicate that the high-order method achieves superior drag reduction in large scale airfoil optimization while maintaining high parallel efficiency. This work marks a successful application of the high-order DDG method to large-scale 3D shape optimization, thereby providing a feasible pathway for its practical implementation.

P24-1 Relativistic Flows

Relativistic Navier-Stokes models

Freistühler, Heinrich

University of Konstanz, Germany

Thursday, May 28, 16:00-16:25, M17.52 – 5.019

Recent proposals model dissipative relativistic fluid dynamics via second-order hyperbolic systems of the form

$$\sum_{j=0}^{d} \frac{\partial}{\partial x_j} \left(F^j(U)\right) = \sum_{j,k=0}^{d} \frac{\partial}{\partial x_j} \left(B^{jk}(U) \frac{\partial U}{\partial x_k}\right), \tag{22}$$

to be solved for functions $U : \mathbb{R}_+^{1+d} \to \mathfrak{U}$ of $0 \leq x_0 \equiv t < \infty, (x_1, ..., x_d) \in \mathbb{R}^d$ where $\mathfrak{U} \subset \mathbb{R}^n$ is given.

Systems (22) contrast with hyperbolic-parabolic systems of conservation laws

$$\sum_{j=0}^{d} \frac{\partial}{\partial x_j} \left(F^j(U)\right) = \sum_{j,k=1}^{d} \frac{\partial}{\partial x_j} \left(B^{jk}(U) \frac{\partial U}{\partial x_k}\right) \tag{23}$$

like classical compressible Navier-Stokes, in which the right-hand side contains no derivatives with respect to t.

This talk compares a family of models originating from [1] with a different family going back to [2].

[1] F. S. Bemfica, M. M. Disconzi, J. Noronha: Causality and existence of solutions of relativistic viscous fluid dynamics with gravity, Phys. Rev. D 98 (2018), 104064.

[2] H. Freistühler, B. Temple: Causal dissipation and shock profiles in the relativistic fluid dynamics of pure radiation. Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. 470 (2014), 20140055, 17 pp.

High order numerical discretizations of the Einstein-Euler equations in the generalized harmonic formulation

Muzzolon, Stefano; Dumbser, Michael; Zanotti, Olindo; Gaburro, Elena

University of Verona, Italy

Thursday, May 28, 16:25-16:50, M17.52 – 5.019

In this talk we propose two new alternative numerical schemes to solve the coupled Einstein-Euler equations in the Generalized Harmonic formulation. The first one is a finite difference (FD) Central Weighted Essentially Non-Oscillatory (CWENO) scheme on a traditional Cartesian mesh, while the second one is an ADER (Arbitrary high order Derivatives) discontinuous Galerkin (DG) scheme on 2D unstructured polygonal meshes. The latter, in particular, represents a preliminary step in view of a full 3D numerical relativity calculation on moving meshes. Both schemes are equipped with a well-balancing (WB) property, which allows to preserve the equilibrium of *a priori* known stationary solutions exactly at the discrete level. We validate our numerical approaches by successfully reproducing standard vacuum test cases, such as the robust stability,

the linearized wave, and the gauge wave tests, as well as achieving long-term stable evolutions of stationary black holes, including Kerr black holes with extreme spin. Concerning the coupling with matter, modeled by the relativistic Euler equations, we perform a classical test of spherical accretion onto a Schwarzschild black hole, as well as an evolution of a perturbed non-rotating neutron star, demonstrating the capability of our schemes to operate also on the full Einstein-Euler system. Altogether, these results provide a solid foundation for addressing more complex and challenging simulations of astrophysical sources through DG schemes on unstructured 3D meshes.

Acknowledgements. S. Muzzolon and E. Gaburro gratefully acknowledges the support received from the European Union with the ERC Starting Grant ALcHyMiA (grant agreement No. 101114995).

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The relativistic Euler equations with Synge energy

Thein, Ferdinand; Ruggeri, Tommaso; Xiao, Qinghua

Johannes Gutenberg-University Mainz, Technical University Clausthal, Germany

Thursday, May 28, 16:50-17:15, M17.52 – 5.019

In this talk we study the relativistic Euler equations together with a constitutive equation which is compatible with the kinetic theory, i.e., with the Synge energy for the monatomic gas and the generalized Synge energy for the diatomic Gas. We show existence and uniqueness of self similar radially symmetric solutions to the initial-boundary problem of the system under consideration. This is the first multi–dimensional result for this system together with the Synge energy. Moreover, in contrast to previous results, due to the properties of the Synge energy, we also cover the ultra relativistic limit, as well as the classical case of the polytropic ideal gas. This is done in a unified treatment and no distinction between the cases is needed.

Constraints preserving Lax-Wendroff flux reconstruction for relativistic hydrodynamics with general equations of state

Basak, Sujoy; Babbar, Arpit; Kumar, Harish; Chandrashekar, Praveen

Indian Institute of Technology Delhi, India

Thursday, May 28, 17:15-17:40, M17.52 – 5.019

In the realm of relativistic astrophysics, the ideal equation of state with a constant adiabatic index provides a poor approximation due to its inconsistency with relativistic kinetic theory. However, it is a common practice to use it for relativistic fluid flow equations due to its simplicity. Here we develop a high-order Lax-Wendroff flux reconstruction method on Cartesian grids for solving relativistic hydrodynamics equations with several general equations of state available in the literature. We also study the conversion from conservative to primitive variables, which depends on the equation of state in use, and provide an alternative method of conversion when the existing approach does not succeed. For the admissibility of the solution, we blend the high-order method with a low-order method on sub-cells and prove its physical admissible property

in the case of all the equations of state used here. Lastly, we validate the scheme by several test cases having strong discontinuities, large Lorentz factor, and low density or pressure in one and two dimensions.

Provably physical-constraints-preserving alternative finite difference-WENO scheme for special relativistic hydrodynamic (RHD) equations

Upadhyay, Shubham; Biswas, Biswarup; Kumar, Rakesh

Mahindra University, India

CANCELLED

We develop a high-order accurate, physical-constraints-preserving alternative finite difference weighted essentially non-oscillatory (AFD-WENO) scheme for the special relativistic hydrodynamic (SRHD) equations. The proposed method employs the local Lax–Friedrichs flux splitting, WENO interpolation, a physical-constraints-preserving flux limiter, a physical-constraints-preserving state limiter, and strong stability-preserving time discretization.

An AFD-WENO scheme with physical-constraints preservation for hyperbolic systems was previously developed by Balsara *et al.* (2025, "Alternative Finite Difference WENO-Like Scheme with Physical Constraint Preservation for Divergence-Preserving Hyperbolic Systems"). In this work, we rigorously extend and apply this framework to the SRHD system. We emphasize the necessity of employing a state limiter in addition to the flux limiter to guarantee the preservation of admissible physical states.

Numerical experiments in one and two space dimensions demonstrate that the proposed scheme achieves high-order accuracy and robustly preserves physical constraints for a variety of standard relativistic flow test problems, where violations of physical constraints frequently occur.

Acknowledgements. The work of Biswarup Biswas is supported by ANRF, Govt. of India grant with file No. SUR/2022/001786.

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[2] D. S. Balsara, D. Bhoriya, and C.-W. Shu, An alternative finite difference WENO-like scheme with physical constraint preservation for divergence-preserving hyperbolic systems, *Communications on Applied Mathematics and Computation*, (2025). https://doi.org/10.1007/s42967-025-00517-y

P25-1 Machine Learning Methods

Stochastic collocation for the Euler system and neural networks

Schneider, Simon; Lukáčová-Medviďová, Mária

Johannes Gutenberg University Mainz, Germany

Thursday, May 28, 14:00-14:25, M17.02

We investigate the convergence of a stochastic collocation method for the compressible Euler system with random initial data. The method can be used without prescribing the probability distribution of the initial data. Our analysis is performed under the hypothesis of boundedness in probability of the discrete derivatives of the numerical approximations. We discuss this hypothesis and possible generalizations for similar hypotheses. Implications for the Navier-Stokes system are discussed as well. Numerical tests illustrate our theoretical findings. We also discuss how neural networks and physics-informed learning can be combined with the method.

We acknowledge the financial support of SPP 2410 "Hyperbolic Balance Laws: Complexity, Scales and Randomness", TRR 146, the Mainz Institue of Multiscale Modeling and the Gutenberg Research College.

Hybrid modeling of hyperbolic systems of equations and auto-differentiability

Després, Bruno

Sorbonne University, LJLL, France and MEGAVOLT, Inria, France

Thursday, May 28, 14:25-14:50, M17.02

Machine Learning [1] is the new revolution in applied sciences. A recent major breakthrough [2] is meteorology modeling [3] with hybrid models. Hybrid models are based on the coupling of a standard PDE fluid model with a Neural Network representation of sources. The Neural Network part is trained with respect to meteorology databases.

A possible model for fluid flow around the earth can be inferred from [2, 3]. It is a set of Euler/Boussinesq/St Venant-like system of equations with Coriolis force (in abstract form)

$$\begin{cases} \partial_t U + \mathcal{L}(U, \partial_x U, \partial_{xx} U, \dots) = S_\theta^{NN}(\alpha), & \theta = \text{all Neural Network weights}, \\ U(x,0) = \beta, & \text{initial data at time } t = 0, \end{cases} \tag{24}$$

where one has a large dataset of observations $\mathcal{D} = \{(\alpha_i, \beta_i, \gamma_i) \mid 1 \le i \le N \text{ with } 1 \ll N\}$. The initial solution is β, the current observations are α and the final solution is γ. The Neural Network modeling of the sources $S_\theta^{NN}(\alpha)$ can be reinterpreted as a local subgrid model. A cost function evaluated at given final time $T > 0$ is written as

$$J(\theta) = \frac{1}{N} \sum_i \int_{x \in \Omega} |U(x, T : \alpha_i, \beta_i, \theta) - \gamma_i|^2 \, dx \qquad (+ \text{ many other terms}).$$

The function J must be auto-differentiable with respect to the weights θ so that one can train/minimize it and obtain a quasi-optimal value θ_{opt}. The critical mathematical issue [3] is that the discrete solution $U_{h,\Delta t}(x, n\Delta t : \alpha_i, \beta_i, \theta)$ of the hyperbolic model must be auto-differentiable as well.

A simple example is the 1D advection equation $\partial_t u + a\partial_x u = 0$ with the upwind scheme $\frac{u_j^{n+1} - u_j^n}{\Delta t} + a \frac{u_{j+\frac{1}{2}}^n - u_{j-\frac{1}{2}}^n}{\Delta x} = 0$. The flux $u_{j+\frac{1}{2}}^n = G_a(u_j^n, u_{j+1}^n)$ follows the characteristic lines: $G_a(\alpha, \beta) = \alpha$ if $a > 0$ and $G_a(\alpha, \beta) = \beta$ if $a \le 0$. The time steps are composed one after the

other. What matters is the regularity of the flux where $R(x) = \max(0, x)$ is a notation for the ReLU function

$$F(a, \alpha, \beta) = aG_a(\alpha, \beta) = R(a)\alpha - R(-a)\beta.$$

The observation is that the upwind scheme, and so any numerical scheme which follows the characteristic lines, contains the ReLU function which evidently is not differentiable at the origin.

Therefore auto-differentiability of discrete hyperbolic models is an open problem per se, even for the calculation of strong solutions. I will review some recent mathematical and numerical advances [4, 5] which establish a functional setting for auto-differentiability of the upwind scheme and, more generally, for auto-differentiability of discrete hybrid models of hyperbolic models.

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[2] Kochkov et al. Neural general circulation models for weather, Nature, 2024.

[3] Brenner, The Neural GCM and other remarks, online, College de France, 2025.

[4] Després. A framework for nonsmooth autodiff with maxpooling functions, TMLR, 2025.

[5] Després, The limit set of Neural Network functions trainable by Machine Learning, in prep.

Failsafe continuous Galerkin discretizations with stabilizations based on artificial intelligence

Hajduk, Hennes

TU Dortmund University, Germany

Thursday, May 28, 14:50-15:15, M17.02

High-resolution schemes for hyperbolic problems rely on adaptively combining a high-order target scheme with a property-preserving low order method. The former is meant to be used where the solution is smooth to preserve accuracy while the latter is required, e.g., at shocks to avoid problems such as Gibbs phenomena. In addition, physical constraints including nonnegativity of certain quantities of interest (either conserved or derived unknowns) may need to be enforced in computational schemes. In this work, we distinguish between numerical admissibility (shock-capturing) and physical admissibility and enforce the latter with established failsafe limiting techniques. Linear and/or nonlinear stabilization can be achieved in a multitude of ways but here we focus on a purely data-driven approach. Using a dissipative WENO formulation for stabilizing discretizations of hyperbolic conservation laws, we train an artificial intelligence (AI) model to produce high-order target solutions in smooth regions, while incorporating sufficient amounts of numerical viscosity in the vicinity of shocks. To avoid failures of the AI-based scheme, we subsequently enforce all nonnegotiable constraints via algebraic flux correction (AFC) schemes. In particular, we ensure the validity of global maximum principles for scalar conservation laws and nonnegativity of density and internal energy for the Euler equations. A main novelty of our AI-based property-preserving approach is that we employ high-order continuous Galerkin methods as underlying discretizations. This is made possible by adding the dissipative terms to antidiffusive fluxes or element contributions and limiting via the AFC framework.

P25-2 Machine Learning Methods

Machine learning–enhanced junction modeling for shallow-water channel networks

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Thursday, May 28, 16:00-16:25, M17.02

The dynamics of water flow through an irrigation channel network can be described by the solutions of the two-dimensional shallow-water equations (Saint-Venant equations) within a 2D geometric framework representing both the the incoming/outgoing channels and their junction regions. Computation becomes more efficient if the dynamics are assumed to be homogeneous across the channel cross-sections, allowing the network geometry to be represented as a set of one-dimensional intervals coupled at junction points. Along each 1D interval, the water dynamics are governed by the one-dimensional shallow-water equations. However, a major challenge of this approach lies in defining appropriate boundary conditions at the junction points to couple the water height and velocity across the different intervals. Continuity of water energy is often imposed at the junctions, but such models frequently neglect the specific geometry of the junctions, including channel cross-sections and the angles between them.

We propose a machine learning–based approach to predict the boundary 1D water heights and velocities near channel junctions. The learning framework relies on 2D shallow-water solutions computed for various channel cross-sections, junction angles, and initial conditions. From these solutions, data are extracted near the junction points to construct suitable junction representations. A neural network is then trained to learn the water dynamics at the junction. The overall irrigation channel dynamics are subsequently described by coupling the neural network predictions at the junctions with a 1D shallow-water solver applied to the incoming and outgoing channels. Simulation results are presented and compared with existing approaches.

A structure-preserving numerical method based on a neural network for a hyperbolic relaxation system

Song, Yingjie

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Thursday, May 28, 16:25-16:50, M17.02

Joint work with **Christian Rohde** (University of Stuttgart)

We consider a hyperbolic relaxation system introduced in [1] that approximates the incompressible Navier–Stokes equation, which is recovered as a singular limit of a hyperbolic system depending on two parameters. This setting leads to a multiscale problem characterized by the coexistence of transport-dominated dynamics and stiff relaxation effects. To learn a proper discretization of the system dynamics, we develop a neural network–based numerical method guided by classical hyperbolic theory and splitting principles. The separation of transport and relaxation dynamics is learned implicitly, without enforcing a predefined splitting scheme. The splitting philosophy is elevated to the network design, with the transport component leveraging the advantages of Fourier Neural Operator (FNO) [2] to ensure physical consistency in the propagation of information along characteristic directions. In addition, the relaxation component exploits the attention mechanism based on the Rotary Position Embedding (RoPE) [3] and the domain of dependence to better respect the space–time causality. This structure-preserving design enables

stable and accurate approximations across different regimes and consistently captures the incompressible Navier–Stokes limit without explicitly enforcing divergence-free constraints. Numerical experiments demonstrate that the proposed method yields accurate and stable approximations across different regimes.

[1] Q. Huang, C. Rohde, W.-A. Yong, and R. Zhang. A hyperbolic relaxation approximation of the incompressible Navier–Stokes equations with artificial compressibility. *Journal of Differential Equations*, 438:113339, 2025.

[2] Z. Li, N. B. Kovachki, K. Azizzadenesheli, B. Liu, K. Bhattacharya, A. M. Stuart, and A. Anandkumar. Fourier neural operator for parametric partial differential equations. *arXiv preprint* arXiv:2010.08895, 2020.

[3] J. Su, Y. Lu, S. Pan, A. Murtadha, B. Wen, and Y. Liu. *RoFormer: Enhanced transformer with rotary position embedding.* arXiv:2104.09864, 2021.

A fully-learnable PDE-informed framework for conservation laws

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Thursday, May 28, 16:50-17:15, M17.02

Physics-Informed Neural Networks (PINNs) represent an important paradigm for the numerical approximation of partial differential equations (PDEs) by integrating physical laws directly into the loss function of deep learning models. It is well-known (see, e.g., [4]) that nonlienar hyperbolic conservation laws pose several challenges characterized by the formation of *shock waves* and *nonlinear interaction of rarefaction fans*, i.e., the solution is no longer smooth, and multiple weak solutions may satisfy the same initial data. To ensure uniqueness and physical validity, one must identify the entropy solution [4]. This poses additional challenges for their numerical solution and, in particular, in the PINNs' framework either 1D or Multi-D (see, e.g., [5, 6]).

Building upon recent works involving hybrid weak PINN formulations in 2D [1, 3, 2], this research proposes a novel, fully-learnable PDE-informed framework that extends the classical PINN architecture to multi-dimensional conservation laws. The central innovation of this work is the departure from fixed, analytical entropy functions. Instead, we introduce a secondary, coupled neural network $\mathcal{N}_\eta$ designed to learn an optimal entropy-entropy flux pair (η, q) tailored to the specific dynamics of the underlying PDE. The primary network approximates the solution $u(x, y, t)$, while the secondary network learns the entropy $\eta(u)$ and its corresponding flux $q(u)$. This pair is governed by the consistency condition $\nabla_u q(u) = \nabla_u \eta(u) \cdot \nabla_u H(u)$, where $H(u)$ denotes the physical flux vector. To guarantee mathematical admissibility, the framework enforces C^2 regularity and strict convexity on $\eta(u)$. This is achieved through a specialized loss component that penalizes the negative eigenvalues of the Hessian matrix $\nabla^2\eta$. By ensuring $\nabla^2\eta > 0$, we satisfy the fundamental requirement for entropy stability in hyperbolic systems.

In this work, we combine new insights and aspects of machine learning based on a fully-learnable entropy framework for the numerical treatment of hyperbolic conservation laws.

[1] Eduardo Abreu and Joao B Florindo. A study on a feedforward neural network to solve partial differential equations in hyperbolic-transport problems. In *Computational Science–ICCS 2021: 21st International Conference, Krakow, Poland, June 16–18, 2021, Proceedings, Part II 21*, pages 398–411. Springer, 2021.

[2] Eduardo Abreu, João Batista Florindo, and Wanderson Lambert. A pde-informed and physics-informed neural network for learning weak solutions in two-dimensional conservation laws. *9th European Congress on Comput. Meth. in Applied Sci. & Engineering*, 2024.

[3] Rafael A. F. Carniello, João Batista Florindo, and Eduardo Abreu. A pinn computational study for a scalar 2d inviscid burgers model with riemann data. *8th European Congress on Computational Methods in Appl. Sciences & Engineering*, 2022.

[4] Constantine M Dafermos. *Hyberbolic conservation laws in continuum physics (325, 4th ed. 2016 Edition)*. Springer, 2005.

[5] Tim De Ryck and Siddhartha Mishra. Numerical analysis of physics-informed neural networks and related models in physics-informed machine learning. *Acta Numerica*, 33:633–713, 2024.

[6] Tim De Ryck, Siddhartha Mishra, and Roberto Molinaro. wpinns: Weak physics informed neural networks for approximating entropy solutions of hyperbolic conservation laws. *SIAM Journal on Numerical Analysis*, 62(2):811–841, 2024.

P26-1 Symmetry Methods

A unified symmetry framework for an Aw–Rascle model with Chaplygin gas dynamics

Maurya, Sandhya; Hota, Tapan Kumar

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Thursday, May 28, 14:00-14:25, M17.52 – 5.019

This study presents a unified symmetry-based analysis of a modified Aw–Rascle system with a Coulomb-like friction term under three Chaplygin-type equations of state: Classical, Generalized, and Modified Chaplygin gas. The model constitutes a nonlinear hyperbolic balance law with nonconservative source terms, relevant to traffic flow and granular dynamics. For each pressure law, a systematic local Lie symmetry analysis is performed, yielding exact invariant solutions that describe self-similar density evolution, friction-induced acceleration, and smooth rarefaction regimes.

For the Classical Chaplygin gas, inverse potential systems associated with the admitted symmetries are constructed. These uncover hidden conservation structures and lead to nonlocal exact solutions capturing spatial localization, density accumulation, and congestion patterns. In the frictionless limit, traveling-wave reductions are derived to recover coherent nonlinear wave solutions that are destroyed by explicit time-dependent forcing. The resulting analytical waveforms include stop–go waves and compression–rarefaction fronts, clarifying the intrinsic nonlinear wave mechanisms governed by Chaplygin-type pressure laws.

The physical admissibility of all solutions is examined through density positivity and bounded velocity fields. A comparative analysis highlights the role of the equation of state in regulating dissipation, wave propagation, and relaxation. The proposed framework integrates symmetry analysis, inverse potentials, and exact wave solutions to provide analytical insight into nonlinear hyperbolic dynamics relevant to continuum traffic and granular flow models.

Fast reconnection for incompressible MHD equations

Ciampa, Gennaro

Università degli Studi dell'Aquila, Italy

Thursday, May 28, 14:25-14:50, M17.52 – 5.019

In this talk, we study magnetic reconnection, understood as a change in the topology of magnetic field lines, for sufficiently regular solutions of the three-dimensional incompressible magnetohydrodynamic (MHD) equations. We construct a class of initial data for which reconnection occurs on time scales strictly shorter than the resistive diffusion time scale, despite the persistence of regularity. This separation of time scales is driven by enhanced dissipation induced by the advective dynamics, yielding the first rigorous examples in which advection, rather than resistivity alone, plays a genuinely active role in the reconnection process.

[1] G. Ciampa and R. Lucà. Accelerated and fast magnetic reconnection through enhanced resistive dissipation for MHD equations, arXiv:2510.06801, 2025.

Symmetry analysis of a transport-enhanced blood flow model

Panda, Debendra Prasad; Pandey, Manoj

BITS Pilani, K K Birla Goa Campus, Goa, India

Thursday, May 28, 14:50-15:15, M17.52 – 5.019

This work presents a comprehensive investigation of one-dimensional blood flow and transport dynamics in arteries and veins. Employing Lie symmetry analysis, we construct an optimal one-dimensional subalgebra and derive exact solutions for the governing equations. Furthermore, we develop a systematic framework to generate an interconnected network of nonlocally related partial differential equation systems. These systems are built from local conservation laws and symmetry-based methods, yielding nonlocal symmetries that are classified and subsequently used to obtain additional exact solutions. As an application, one of these solutions is employed to analyze the evolution of weak discontinuity waves. The results provide new insights into circulatory dynamics and indicate promising applications for advanced hemodynamic modeling and analysis.

[1] A. Spilimbergo, E. F. Toro, A. Siviglia and L. O. Müller. Flux vector splitting schemes applied to a conservative 1D blood flow model with transport for arteries and veins. *Computers & Fluids*, 271:106165, 2024.

[2] P. J. Olver. Applications of Lie Groups To Differential Equations. Springer-Verlag New York, 1993.

[3] G. W. Bluman and A. F. Cheviakov. Framework for potential systems and nonlocal symmetries: Algorithmic approach. *Journal of Mathematical Physics*, 46:123506, 2005.

Semi-analytical solution of $(2+1)$-dimensional KdV-Burgers equation using homotopy analysis method

Arora, Rajan; Sharma, Prince

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Thursday, May 28, 15:15-15:40, M17.52 – 5.019

This work presents a semi-analytical investigation of the $(2+1)$-dimensional Korteweg-de Vries-Burgers (KdV-Burgers) equation using the Homotopy Analysis Method (HAM). The equation describes nonlinear wave propagation in dispersive-dissipative media and serves as an important model for studying the combined effects of nonlinearity, dispersion, and dissipation. The HAM framework is employed to construct a convergent series solution without the requirement of small perturbation parameters, while providing explicit control over convergence through an auxiliary parameter. An appropriate linear operator and homotopy deformation structure are formulated to systematically generate higher-order approximation terms under prescribed initial conditions. The convergence properties of the obtained series solution are quantitatively analyzed using the averaged squared residual error technique, allowing the determination of an optimal convergence-control parameter. The resulting HAM-based solution exhibits excellent agreement with the corresponding exact traveling-wave solution, as verified through numerical error analysis and graphical comparison. The results accurately reproduce smooth nonlinear wave profiles and demonstrate the effectiveness of HAM as a reliable semi-analytical tool for analyzing higher-dimensional nonlinear evolution equations arising in dispersive-dissipative wave phenomena.

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